Optimal law enforcement with multiple criminal organizations

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Abstract

This paper develops a model with regard to the optimal law enforcement on organized crime with multiple criminal organizations (the Mafias) which regulate the criminal market by extortion. It enables us to consider strategic interaction relationships among the criminal organizations and the government. This paper shows that welfare effects of introducing multiple criminal organizations depends on how they face a competition in this criminal market. If each criminal organization faces a fierce competition, it is hard for criminal organizations to make profits and play the role as a regulator. On the other hand, if there exists no competition among criminal organizations, introducing multiple criminal organizations contributes to the social welfare improvement. This indicates that the classical view stressing the desirable effects of monopolistic criminal organization is not always supported. Furthermore, by considering the costly conflicts among criminal organizations, this paper also explores whether harsh penalties on the criminal market induce them to engage in conflicts and shows that it can be difficult to sustain the desirable criminal markets regulated by multiple criminal organizations.

keywords: Conflict, Oligopoly, Organized Crime

JEL Classification: D74, K4, L13,
1 Introduction

The literature on the economics of crime has focused on the rational individual’s decision problem as regards whether to engage in criminal activities. Such an analysis was originally started with Becker’s (1968) seminal work. Based on the Becker’s approach, a lot of researchers have tried to examine what is the optimal deterrence policy the government should take and its effect on social welfare and efficiency. See Garoupa (1997) and Polinsky and Shavell (2000) for an overview.

Even if organized crime is also an important issue for economics, the economic analysis of organized crime is still scarce as pointed out by Fiorentini and Peltzman (1995). However, there exist also some important studies about organized crime. Almost previous papers have stressed the welfare comparisons between monopolistic criminal markets (with a criminal organization) and competitive criminal markets (without criminal organization) because a criminal organization is thought of a monopolistic firm, i.e., Schelling (1971), Buchanan (1973), Gambetta and Reuter (1995) and Garoupa (2000)\(^1\). According to such a monopolistic view, the monopolistic criminal market is desirable than competitive criminal market because social bads, like criminal activities, supplied by a criminal organization become smaller.

However, such a monopolistic view on a criminal organization is true? In reality, we often observe miserable conflicts among criminal organizations as in Japan, Italy and other countries. Considering such an actual affair, focusing on only monopolistic markets provides limited meaningful implications. Also, Fiorentini (1995) argues that there exists no convincing reason to support the monopolistic view of illegal markets. Therefore, by extending to multiplicity of criminal organizations, this paper tries to give an important and new insight into deterrence policies against them. Furthermore, we also aim to examine whether the multiplicity of criminal organization is beneficial for the social welfare and efficiency.

Motivated by these observations, apart from the monopolistic view, we examine the oligopolistic criminal market controlled by multiple criminal organizations and its effect on optimal law enforcement the government takes. We extend the optimal law enforcement with a monopolistic criminal organization originally proposed in Garoupa (2000) to incorporating the multiplicity of criminal organizations. Following Garoupa, the role of criminal organizations is to regulate the criminal market by extortion. This means that potential offenders must buy a license from them to commit an illegal act\(^2\).

Since the main difference between a monopoly and an oligopoly is intro-


\(^2\)There exist some other papers extending Garoupa (2000) to different directions. One of them is Chang et al. (2005) incorporating the possibility of coexisting of individual criminals and organized crime. The role of criminal organization is the same, but potential offenders can choose whether to be a member of criminal organization. Another paper is Garoupa (2007) which focuses on internal organization aspects between the principal and agents.
ducing a strategic interaction, we consider different two types of competition. The first competition is to attract potential offenders in entering the criminal markets. If each criminal organization can differentiate their criminal markets, competition among them may occur. On the other hand, if each criminal organization does not need to differentiate, they face no competition. Hence, we model these two different competition structures as regards to the first type competition.

The second competition is to acquire the monopolization by engaging in conflicts with violence. As observed in reality, conflicts for market opportunities among criminal organizations are constantly continuing\(^3\). Such an incentive for waging in wars is derived by the benefits from obtaining the monopoly profits. Thus, if criminal organizations prefer the monopoly profits to the oligopoly profits, conflicts will be emerged.

By incorporating these two feature, this paper gives new insights which have not been pointed out before. First, this paper shows that the classical view stressing the superiority of a monopolistic criminal organization does not always hold. Whether to hold such a classical view depends on how the strategic relationship among criminal organizations is. If criminal organizations face completely fierce competition in collecting potential offenders, oligopolistic criminal markets can not be sustained because criminal organizations can not obtain profits and, as a result, the superiority of introducing criminal organizations as stressed in a monopoly case is not achieved. Hence, each criminal organization has an incentive to wage wars and, as a result, monopolizes criminal markets. This result is very intuitive. This indicates that the government must account for the extra cost caused in conflicts, so the optimal law enforcement may become harsher and the expenditure on enforcing law also increases. This result contrasts to the one proposed in Garoupa (2000) which stresses a criminal organization contributes to less harsh deterrence policy. On the other hand, if criminal organizations face no severe competition like above, oligopolistic criminal markets will contribute to improvement of the social welfare efficiency. Even if this result reinforces the superiority of introducing criminal organizations, but this does not support the desirability of the monopolistic criminal market. However, even if an oligopolistic criminal market is desirable for the society, criminal organizations with strong military power prefer conflicts to peace. This might cause social welfare loss as discussed in the above case and duopolistic market cannot be sustained.

Second, this paper models the violence and its effect in an explicit way. For this framework, we can obtain the analytical strength as regards to the interaction between the incentive to wage wars and the enforcement of law. It is unclear whether the harsh penalty by the government causes more conflicts. This paper gives some explanations to this complicated problem and shows that harsher penalties do not always bring about conflicts. Our framework has common with Castillo (2015), but the motivation is different because Castillo

\(^3\)Hill (2004) examines that how inter Japanese criminal organizations disputes arise and exhibits some resolutions and difficulties. Also, Catanzaro (1994) also argues the conflicts among Italian criminal organizations.
focuses on how the peaceful equilibrium can be achieved in a repeated game setting. Furthermore, the role of criminal organizations is different from our settings as a regulator of criminal markets.

There exist some other papers focusing on the economic effects of oligopolistic criminal organizations in bads markets. Fiorentini (1995) models the illegal goods market, like drugs, and considers the oligopolistic competition in price and quantities. Fiorentini shows that whether a monopolization contributes to the reduction of social bads depends on the type of government’s strategy allowing the biased deterrence policy against some organizations. Masour et al. (2006) examines the model in which a group formation of criminal organizations is endogenous without the use of violence. Kugler et al. (2005) considers the possibility of corruption between the law enforcer and multiple criminal organizations.

This paper organizes as follows. Section 2 describes the basic model of the optimal law enforcement. In section 2.1, we introduce the benchmark result in case of without criminal organization which was originally proposed in Garoupa (2000). Section 3 introduces the model developed by Garoupa and consider our extensions. Focusing on the different form of strategic relations, we derive the optimal strategy each economic actor should take in section 3.1 and 3.2. In section 4, we add the possibility of engaging in conflicts and its effects in a duopoly market case. Finally, section 5 concludes.

2 A model without criminal organization

In this section, we introduce the basic model on the optimal enforcement of law which is identical to Garoupa (2000). We introduce risk-neutral potential offenders choosing whether to commit an illegal act that benefits the offender by $b$, which vary across potential offenders, and harms the rest of society by $h > 1$. The government does not know any offenders’ $b$ but knows its distribution: $b$ is uniformly distributed over $[0, 1]$. The assumption $h > 1$ means that committing an illegal act is not socially beneficial.

The government chooses a sanction $f$ and a detection and conviction probability $p$. The expenditure on detection and conviction is $C(p)$, which is increasing function of $p$. For simplicity, we assume $C(p) = cp$ where $c > 0$ is a cost parameter. Also, we assume $0 \leq f \leq F$ where $F$ is the maximum feasible sanction, which can be interpreted as the maximum wealth of individuals. The objective function of the government is to maximize the social welfare: the sum of offenders’ benefits minus the harm caused by them and enforcement cost. We followed the standard assumption that offenders’ benefits are taken into consideration for the social welfare.

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4 Castillo (2015) considers the possibility of conflicts among drug trafficking cartels. Castillo models violence in an explicit way and examines the incentives of cartels to wage a war or stay in peace. Castillo shows that there can be an oligopolistic market in equilibrium and such an oligopolistic and peaceful equilibrium also contributes to the reduction of violence.
2.1 A competitive criminal market

This subsection introduces the main result proposed by Garoupa (2000). As an important benchmark, we consider a no criminal organization situation. We call it a competitive criminal market because, as we mentioned in introduction, there exists no criminal organization which can regulate to enter this criminal market. Potential offenders commit an illegal act if and only if $b \geq pf$. Thus, the social welfare is

$$W = \int_{pf}^{1} (b - h)db - cp$$

The government maximizes the social welfare $W$ with respect to $p$ and $f$. Let us use the subscript $C$ to denote the results obtained in a competitive criminal market.

Proposition 1 (Garoupa (2000)). In a competitive criminal market, the optimal fine is the maximum fine ($f = F$). The optimal detection probability is $p^C F = h - c/F$.

3 A model with multiple criminal organizations

In this section, we introduce criminal organizations (the Mafias). The role of the Mafia is to regulate the criminal market by extortion. This means that potential offenders must buy a license from the Mafia to commit an illegal act. Garoupa (2000) treats criminal organizations as a vertical structure: the Mafia extracts some rents from potential offenders with extortion. This paper aims to extend the Garoupa’s model to an oligopolistic competition among multiple criminal organizations by incorporating the strategic aspects. Formally, there exist the $n$ Mafias and each potential offender must pay $e_i$ to one of the Mafia $i \in \{1, 2, ..., n\}$ to commit an illegal act. The Mafia $i$ tries to maximize profits with respect to $e_i$.

What is the strategic relationship among the Mafias? We introduce different two types of competition: (a) competition for attracting potential offenders and (b) competition for the monopolization. The type (a) competition indicates that, for example, if Mafia 1 demands an excessive license fee and Mafia 1 demands no license fee, potential offenders would want to enter the latter criminal market. In this case, as in the price competition like the Bertrand competition, demanding a high price will cause the low profit for the Mafia. However, is there always such a fierce competitions among the Mafias? If potential offenders can not move across each district, severe competition will not arise. Also, collecting information about the mafia may be difficult for potential offenders and there

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5 This paper will consider only costless extortion situations although Garoupa also considers extortion is costly.
may be no competition among the Mafias. Therefore, as for (a) competition, we consider further different two types of strategic relations among the Mafias: (a.1) no competition and (a.2) Bertrand competition.

As for type (b) competition, if the Mafias find the profit in a monopolistic criminal market is larger than in an oligopolistic criminal market, the Mafia may have an incentive to wage a war even if a war takes a cost. In this case, the government must take care of the social welfare loss caused in conflicts. Thus, the strategy the government should take will be changed. At first, we consider only (a.1) and (a.2) competition in section 3.1 and 3.2. The type (b) competition will be considered in section 4.

3.1 A criminal market with multiple criminal organizations: no competition case

If there exists no competition among the Mafias, the potential offenders would enter the criminal markets controlled by Mafia $i$ with some probability $q_i$. For simplicity, we assume that $q_i = 1/n, \forall i$. Thus, the expected benefit for potential offenders from committing an illegal act is $b - pf - \sum e_i/n$. Thus, potential offenders enter the criminal market if and only if $b \geq pf + \sum e_i/n$. Also, the profits of the Mafia $i$ are

$$\pi_i = \int_{pf + \sum e_i/n}^1 db.$$

Following Garoupa (2000), we consider two types of game: (1) Nash-Cournot game and (2) Stackelberg game. In (1), the government and the Mafias decide their strategies simultaneously. In (2), only the government moves first and all the Mafias move later.

Nash-Cournot game. The social welfare function is

$$W = \int_{pf + \sum e_i/n}^1 (b - h)db - cp.$$

The government maximizes the social welfare function with respect to $p$ and $f$ subject to $0 \leq f \leq F$. According to the standard maximization problem with Lagrangean method, the optimal fine is also maximum fine as in Proposition 1. Hence, we obtain the government’s reaction function against the Mafias’ strategies:

$$pf = h - \sum e_i/n - c/F.$$

Also, according to the standard maximization problem, the Mafia $i$’s reaction function against the government strategy are:

$\text{6Garoupa (2000) provides the reasons to take these two approaches.}$
\[ e_i = \frac{n(1 - pf) - \sum_{j \neq i} e_j}{2}. \]  

(5)

Moreover, because all the Mafia have the same objective function, we assume the symmetric equilibrium, so \( e_i = e_j, \forall \ i \ and \ j \). In the following analysis, we assume the interior solutions.

**Proposition 2.** In no competition (NC) case with multiple criminal organizations and a Nash-Cournot game situation, the optimal fine is the maximum fine \( f = F \). As given the number of the Mafias, the optimal detection probability is \( p_{NC}^F = (n + 1)h - n - (n + 1)c/F \). \( p_{NC}^F \) decreases as \( n \) increases.

**Proof.** As already mentioned above, we use the Lagrangean method. Basically, we follow the way of proof as in Garoupa (2000). Define the Lagrangean as \( L = W + \lambda(F - f) \) where \( \lambda \) is the Lagrangean multiplier. The first order conditions are

\[ \frac{\partial L}{\partial f} = p(h - pf - \sum_{i}^{n} e_i/n) - \lambda = 0 \quad \text{and} \quad L_f = p(h - pf - \sum_{i}^{n} e_i/n) - \lambda = 0 \]  

(6)

\[ \frac{\partial L}{\partial p} = f(h - pf - \sum_{i}^{n} e_i/n) - c = 0. \]  

(7)

Suppose that the optimal fine, \( f^* \) is not maximal. From (6), we must have \( h - p^*f^* - \sum_{i}^{n} e_i/n = 0 \) where \( p^* \) is optimal detection probability. However, this is impossible according to (7). Hence, the optimal sanction \( f^* \) must be maximal, \( f^* = F \), and \( \lambda^* > 0 \). Also, \( dp_{NC}^F/dn = h - 1 - c/F < 0 \) because we assume \( 0 < p_{NC}^F = h - c/F < 1 \). Q.E.D.

Note that in case of \( n = 0 \), \( p_{NC}^F = h - c/F = p_C^F \) and in case of \( n = 1 \), \( p_{NC}^F = 2h - 1 - 2c/F \). In particular, the latter results correspond to the monopolistic criminal market proposed in Garoupa (2000). From a theoretical view point, this result can be interpreted as the generalization of the optimal law enforcement with organized crime originally proposed by Garoupa. This result indicates that as the number of the Mafias increases, the government would reduce the expenditure of the enforcement of law. Also, the equilibrium criminal rate and the equilibrium profits for the Mafias do not depend on \( n \).

What is the effects of multiple criminal organizations on social welfare? The answer is summarized as follows:

**Corollary.** In no competition case with multiple criminal organizations and a Nash-Cournot game situation, the equilibrium social welfare is \( W_C^F < W_{n=1}^{NC} < W_{n=2}^{NC} < \ldots < W_{n=n}^{NC} \).

This means that social welfare will improve as the number of Mafias increases. The equilibrium extortion by the Mafias \( e_{NC}^F = n(1 - h + c/F) \) in-
creases with \( n \). This means that each Mafia tries more extortion because of rivalry among the Mafias and, as a result, it is difficult for offenders to enter the market. Thus, criminal market controlled by oligopolistic mafias can contribute to the social welfare efficiency compared to the monopolistic market. Therefore, the classical view about organized crime claiming the desirability of monopolistic bads markets is not supported as long as potential offenders can not choose markets freely.

**Stackelberg game.** In the Stackelberg game situation, the government moves first and the Mafias moves later. Thus, the first order conditions for the Mafia \( i \)'s extortion are \( e_i = n(1-pf) - \sum_{j \neq i} e_j \). As in Nash-Cournot game, we assume the symmetric equilibrium. Thus, the social welfare function is

\[
W = \int_{n/(n+1)+pf/(n+1)}^{1} (b - h) db - cp. \tag{8}
\]

Therefore, according to the standard optimization problem with Lagrangean method, we have the results below.

**Proposition 3.** In no competition case with multiple criminal organizations and a Stackelberg game situation, the optimal fine is the maximum fine \( f^* = F \). As given the number of the Mafias, the optimal detection probability is \( p^{NC}F = (n+1)h - n - (n+1)^2c/F \). \( p^{NC}F \) decreases as \( n \) increases.

**Proof.** By the same argument as in proof of Proposition 2, define the Lagrangean as \( L = W + \lambda(F - f) \) where \( \lambda \) is the Lagrangean multiplier. The first order conditions are

\[
L_f = p(h - pf - n/(n+1) + pf/(n+1)) - \lambda = 0 \quad \text{and} \quad (9)
\]

\[
L_p = f(h - pf - n/(n+1) + pf/(n+1)) - c = 0. \quad (10)
\]

Suppose that the optimal fine, \( f^* \) is not maximal. From (9), we must have \( h - pf^* - n/(n+1) + pf/(n+1) = 0 \) where \( p^* \) is optimal detection probability. However, this is impossible according to (10). Hence, the optimal sanction \( f^* \) must be maximal, \( f^* = F \), and \( \lambda^* > 0 \). \( p^{NC}F \) decreases as \( n \) increases as in the proof in Proposition 2. Q.E.D.

As in Proposition 2, as the number of Mafias increases, the expected punishment \( p^{NC}F \) decreases. The main difference between Nash and Stackelberg game is that extortion will be high in Stackelberg game. The equilibrium crime rate decreases and the profits increase as the number of the Mafias increases. Thus, it is not clear whether the social welfare improves in this case.
3.2 A criminal market with multiple criminal organizations: Bertrand competition case

If a competition among the Mafias is like a Bertrand price competition, the potential offenders would enter the criminal market which gives the highest expected benefits. Therefore, potential offenders enter the criminal market controlled by the Mafia $i \in \{1, 2, \ldots, n\}$ if and only if $b - pf - e_i > b - pf - e_j$ for $j \neq i$. In other words, criminal market $i$ is preferred to any other criminal markets if and only if $e_j > e_i$. This condition is very intuitive. In the same way, we can define the profits for the Mafia $i$ as

$$
\pi_i = \begin{cases} 
\int_{pf+e_i}^{\infty} e_i \, db & \text{if } e_j > e_i, \forall j \neq i \\
\frac{1}{M} \int_{pf+e_i}^{\infty} e_i \, db & \text{if } e_j = e_i \text{ with } |j| = M - 1, \text{ and } e_k > e_i \text{ for other } k \\
0 & \text{if } e_j < e_i, \forall j \neq i.
\end{cases}
$$

In this Bertrand competition case, the Nash Equilibrium is $e^*_i = 0, \forall i$ and $\pi^*_i = 0, \forall i$ in Cournot and Stakelberg game. As a result, potential offenders join the criminal market $i$ if and only if $b \geq pf$. This results means that the profit for the Mafias becomes 0 and there exists no regulation on criminal markets. Therefore, we have the same results as in Proposition 1.

**Proposition 4.** In Bertrand competition (BC) case with multiple criminal organizations, the optimal fine is the maximum fine ($f = F$). The optimal detection probability is $p_{BC} F = h - c/F$. This is the same as in a competitive criminal market.

This proposition says that as long as the Mafias face a fierce competition like the Bertrand price competition, introducing the Mafias will not contribute to improve the social welfare compared to a competitive criminal market case. Therefore, the social welfare improving effects made by the Mafia as in Garoupa (2000) is not applied in oligopolistic criminal markets. Furthermore, the classical view about organized crime claiming the superiority of monopolistic bad markets is also supported as long as potential offenders can choose markets easily.

Remark some extensions. Let us consider that each Mafia can provide the offenders the advantage in committing a crime. That is, the probability of apprehension and conviction varies across the criminal market controlled by the Mafia $i$. We assume that $p_i = p(p, \theta_i)$ is a function of $p$ and the parameter $\theta_i$ differentiating an apprehending probability among the Mafias and is strictly smaller than the apprehending probability in a competitive criminal market, $p_i < p$. This is because the Mafia can provide some information to offenders\textsuperscript{7}. Thus, if we introduce the heterogeneity of apprehending probability, the optimal strategy the government take will be altered.

\textsuperscript{7}Garoupa (2007) examines this problem more deeply.
Motivated by the above extension, we derive another result. Potential offenders enter the criminal market $i \in \{1, 2, \ldots, n\}$ if and only if $b - p_i f - e_i > \max_{j \neq i} b - p_j f - e_j$. Without loss of generality, we assume $p_1 < p_2 < \ldots < p_n$ as given the value of $p$. Entering criminal markets controlled by the Mafia 1 is beneficial for potential offenders as long as every Mafias demand the same license fee. In this case, the Mafia 1 will make use an advantage to gain the profits, so the zero profits equilibrium does not occur as in Proposition 4. In this case, the equilibrium extortion is $e^*_1 = (p_2 - p_1)f - \epsilon$, and $e^*_j = 0 \forall j \neq 1$ where $\epsilon$ is small enough. As a result, the Mafia 1 can obtains the monopolization in this criminal market. As for the government strategy, the government maximize the new social welfare $W = \int b - h db - cp$. This means that the government's strategy becomes less effective, so the government may make less investment in detection. As a result, the equilibrium crime rate increases and it is not clear whether the social welfare improves.

4 An extension: the possibility of conflicts

In this section, we will extend the basic model introduced in section 3 to incorporating the possibility of conflicts among the Mafias. One of the main difference between the legal goods market and illegal goods market is the use of violence. In illegal goods market, the Mafias often resort to violence to exclude other Mafias from this markets and monopolize the rent extracting.

What should be the optimal law enforcement for the government in a criminal market with multiple criminal organizations in case of the possibility the costly conflict? What is the relation between the deterrence policy and the incentive for the Mafias to wage a war? To answer these questions, we integrate the simple conflict theory, i.e., Garfinkel and Skaperdas (2007) and Konrad (2009), into our basic model.

For simplicity, we assume that there exist two Mafias, Mafia 1 and 2. Thus, our setting is a duopolistic criminal market case. The game proceeds as follows. At first, the government announces the detection probability, $p$, and the sanction, $f$. After observing the government strategy, the Mafia 1 and 2 decide whether to wage a war. If there exists no war, the duopolistic profits are realized. If there exists a war, the winner obtains the monopolistc profits and the loser gets no profits. Thus, this setting is an extension of a Stackelberg game situation.

According to our results, monopolistic profits will be $\pi^M \equiv \left(1 - \frac{p_f}{2}\right)^2$. The duopolistic profits in Bertrand competition case (Duopolistic Bertrand Competition) is $\pi^{DBC} = 0$ and in no competition case (Duopolistic No Competition)

8If $e^*_j > 0$ for some $j \neq 1$, potential offender’s expected benefits in criminal market 1 and $j \neq 1$ are $b - p_i f - (p_j - p_1)f + \epsilon$ and $b - p_j f - e_j$. Hence, Mafia 1 can collects all offenders if and only if $\epsilon + e_j + (p_j - p_2)f > 0$. This condition is always satisfied and Mafia $j$’s profit is zero. On the other hand, if $e^*_j = 0$ for all $j \neq 1$, in order for Mafia 1 to obtain profits, Mafia 1 sets $e_1$ such that $b - p_i f - e_1 > \max_{j \neq i} b - p_j f$. Hence, $e_1$ must satisfy $(p_j - p_1)f > e_1$ for all $j \neq 1$. Hence, we must have $e^*_1 = (p_2 - p_1)f - \epsilon$. 

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is $\pi^{DNC} \equiv (1 - \frac{pf}{2})^2$. Based on the basic conflict theory proposed by Tullock (1980), we define the expected profits from engaging in conflict. In case of conflicts, Mafias 1 and 2 can invest military resources $g_1$ and $g_2$ to win conflicts. The winning probability for each Mafias depends on the ratio of the amount of invested resources. Let $p_i$ be the winning probability of the Mafias $i$, thus we have $p_1 = \frac{\beta g_1}{(\beta g_1 + g_2)}$ and $p_2 = \frac{g_2}{(\beta g_1 + g_2)}$ where $\beta \geq 1$. The value $\beta$ represents the relative ability of Mafias 1 in terms of effectivities of investments and we assume that the value is exogenously given. This setting means that if every Mafias invest the same resources, the winning probability of Mafias 1 is greater than Mafias 2. Therefore, the expected profits for Mafias 1 and 2 are

$$\pi_1^W = \frac{\beta g_1}{\beta g_1 + g_2} \pi^M - g_1 \quad \text{and} \quad \pi_2^W = \frac{g_2}{\beta g_1 + g_2} \pi^M - g_2.$$ (12)

The subscript $W$ represents the situation in a conflict and war. Thus, from the standard optimization problem, we have the first order conditions for each Mafias:

$$\frac{d\pi_1^W}{g_1} = \beta g_2 \frac{\pi^M}{(\beta g_1 + g_2)^2} - 1 = 0 \quad \text{and}$$

$$\frac{d\pi_2^W}{g_2} = \beta g_1 \frac{\pi^M}{(\beta g_1 + g_2)^2} - 1 = 0.$$ (13, 14)

Therefore, we have Lemma 1.

**Lemma 1.** The equilibrium invested resources for the Mafias 1 and 2 are $g_1^* = \frac{\beta}{1 + \beta} \pi^M$ and $g_2^* = \frac{1}{1 + \beta} \pi^M$. The equilibrium expected profits for the Mafias 1 and 2 are $\pi_1^W = (\frac{\beta}{1 + \beta})^2 \pi^M$ and $\pi_2^W = (\frac{1}{1 + \beta})^2 \pi^M$.

Hence, in case of Bertrand competition, it is always true that $\pi_1^W > \pi_1^{DBC} = 0$ according to the discussion in section 3.2. Thus, the Mafia has an incentive to avoid a Bertrand competition and try to monopolize a criminal market. Also, in case of no competition, if $\beta > (\leq) 2$, $\pi_1^W > (\leq) \pi_1^{DNC}$. This indicates that as long as the Mafia has an advantage in conflict, the Mafia 1 would like to wage a war. As a result, the social welfare function becomes different from the previous one. Let us denote $r > 1$ as the harm caused in the process conflicts. At first, we examine the no competition case and the social welfare function is summarized below.

$$W^{DNC} = \begin{cases} 
\int_1^{1/2 + pf/2} (b - h) db - (g_1^* + g_2^*) r - cp & \text{if} \quad \beta > 2 \\
\int_2^{1/3 + pf/3} (b - h) db - cp & \text{if} \quad \beta \leq 2.
\end{cases}$$ (15)

\[ ^9 \text{In case of } r = 1, \text{the society suffers from only dissipation, i.e., Nitzan (1991). However, in case of } r > 1, \text{the society suffers from not only dissipation but also harms related with conflicts.} \]
In case of $\beta > 2$, the new social welfare function corresponds to the monopolistic criminal market and contains the social welfare loss caused by conflicts. On the other hand, in case of $\beta \leq 2$, the new social welfare function corresponds to the duopolistic criminal market and no conflict situation. Therefore, we obtain the optimal government strategy.

**Proposition 5.** In no competition case under the possibility of conflict, the optimal fine is the maximum fine ($f = F$). The optimal detection probability is $p^W_F = \frac{2h - 1 - 4c/F + (4\beta r)/(1 + \beta)^2}{(1 + (4\beta r)/(1 + \beta)^2)}$ if $\beta > 2$ and $p^W_F = 3h - 2 - 9c/F$ if $\beta \leq 2$. 

**Proof.** By the same argument as in proof of Proposition 2, define the Lagrangean as $L = W + \lambda (F - f)$ where $\lambda$ is the Lagrangean multiplier. In case of $\beta > 2$, the first order conditions are

$$L_f = p(h/2 - 1/4 - \beta r/(1 + \beta)^2 - pf/4 - pf \beta r/(1 + \beta)^2) - \lambda = 0 \quad \text{and} \quad (16)$$

$$L_p = f(h/2 - 1/4 - \beta r/(1 + \beta)^2 - pf/4 - pf \beta r/(1 + \beta)^2) - c = 0. \quad (17)$$

Suppose that the optimal fine, $f^*$ is not maximal. From (16), we must have $$(h/2 - 1/4 - \beta r/(1 + \beta)^2 - p^* f^*/4 - pf^* \beta r/(1 + \beta)^2) = 0$$ where $p^*$ is optimal detection probability. However, this is impossible according to (17). Hence, the optimal sanction $f^*$ must be maximal, $f^* = F$, and $\lambda^* > 0$. In case of $\beta \leq 2$, this proof follows as in Proposition 3. Q.E.D.

Also, we can derive the optimal strategy in Bertrand competition with duopoly Mafia case.

**Corollary.** In Bertrand competition case under the possibility of conflict, the optimal fine is the maximum fine ($f = F$). The optimal detection probability is $p^W_F = \frac{2h - 1 - 4c/F + (4\beta r)/(1 + \beta)^2}{(1 + (4\beta r)/(1 + \beta)^2)}$. 

This result indicates that if we consider the possibility of conflicts, the expected punishment in a competitive and monopolistic criminal market, $p^C_F$ and $p^M_F$, may be smaller than $p^W_F$ as long as $r$ is large. This means that the introducing the Mafias does not always contribute to the reduction of the expenditure on law enforcement, which contrasts with Garoupa (2000). Even if extortion by the mafia is not costly, multiplicity of the Mafias will induce harsher penalties. Even if the duopoly criminal market is desirable, it cannot be achieved because of difficulties to avoid conflicts.

Let us remark about the value $\beta$ and its effect. What is the equilibrium if each Mafia can choose its effectiveness in conflicts? Let $\beta_H > 2$ and $\beta_L = 1$ be the two values the Mafia can choose. The Mafia has to pay $s$ in obtaining $\beta_H$ and nothing in $\beta_L$. Each Mafia can choose either $\beta_H$ or $\beta_L$ before deciding to engage in conflicts. According to the payoff matrix in Figure 1, choosing $\beta_H$ is optimal for the Mafia 1 and 2 if the cost parameter $s$ is low. This means that if
the Mafia can obtain weapons for conflicts with low cost, the optimal strategy for the Mafias is no conflict and the equilibrium payoff for them becomes smaller. This outcome is similar to the Prisoner’s dilemma. This implicates that if the government does not regulate on the weapons the Mafias use, the low Mafia’s profit can be achieved and there exists no conflict. Hence, in order to achieve low profits of the Mafias, less regulations on the weapons can be optimal. However, in reality, because the government tries to regulate the weapon, so the cost parameter $s$ is high. As a result, no conflict equilibrium can be achieved but the equilibrium profits for the Mafia become high.

In the last part of this section, we consider the heterogeneity of a detection and conviction probability. We assume that a detection and conviction probability in duopolistic market can be different from monopolistic market. If there exists such a heterogeneity, the incentive to engage in conflicts can be differed. Let $p^M = p^M(p)$ and $p^D = p^D(p)$ be the detection probability function in monopolistic and duopolistic criminal market and $p^M$ can be smaller or bigger than $p^D$. Therefore, the expected profits become

\[
\pi^W_1 = (\beta H + \beta L)^2 \pi^M = (\beta H + \beta L)^2 (1 - p^M_f)^2
\]

and

\[
\pi^W_2 = (1 + \beta H)^2 \pi^M = (1 + \beta H)^2 (1 - p^D_f)^2
\]

Also, $\pi^{DNC} \equiv (1 - p^D)^2$ in no competition case. Focusing on no competition case, we examine the incentive for Mafia 1 in four cases: (1) $\beta_H$ and $p^D < p^M$, (2) $\beta_H$ and $p^D > p^M$, (3) $\beta_L$ and $p^D < p^M$ and (4) $\beta_L$ and $p^D > p^M$.

In case of (1) and (2), because the Mafia 1 has a relative advantage in conflicts ($\beta_H$), the Mafia 1 may have an incentive for the war. In case of (1) and (3), a detection and conviction probability in monopolistic market is more effective than one in duopolistic market, $p^M > p^D$. This condition holds, for example, if the government has to allocate resources for each market controlled by the Mafia 1 and 2. On the contrary as in (2) and (4), a detection and conviction probability in monopolistic market is less effective than one in duopolistic market, $p^M < p^D$. This condition holds, for example, if a detection probability depends on the number of offenders. As the number of offenders increases, it is more difficult to detect the offenders.

At first, we examine (2) and (3). In (2), the Mafia 1 always prefers conflicts for any government strategies $p$.\(^{10}\) Therefore, the social welfare function is

\[
W^{DNC} = J_{1/2+p_f/2} (h - h) db - (g_1 + g_2) r - cp.
\]

Also in (3), the Mafia 1 and 2 always prefer no conflict for any government strategies. Therefore, the social

\(^{10}\)This is because $\pi^M = (\frac{\beta}{\beta^2 + \beta})^2 (1 - p^M f)^2$ is always larger than $\pi^{DNC} \equiv (\frac{1}{3})^2 (1 - p^D f)^2$ in case of $\beta_H$ and $p^D > p^M$. 

welfare function is \( W^{DNC} = \int_{1/2}^{1} \left( b - h \right) db - cp. \)

As for (1), since the value of \( \beta \) is high, the Mafia has a relative advantage in conflicts and monopolistic profit is also high in case of low detection probability. However, since \( p^D < p^M \), in case of high detection probability \( p \), duopilistic profits will be high. Thus, there exists the threshold value of a detection probability, \( p^* \), such that if \( p < p^* \), the Mafia 1 prefers war and if \( p > p^* \), Mafia 1 prefers peace. Hence, the new social welfare function is different from the above case\(^{11}\). As for (4), the incentive for causing war is completely different from (1). The low detection probability will make the Mafia 1 cause war and the high detection probability may bring about no conflict equilibrium.

By incorporating the difference of detection probability between a monopoly and a duopoly market, the incentive to wage wars will be varied. We can give the theoretical explanations whether the harsh penalties induce more wars.

5 Concluding remarks

This paper develops a model about the optimal law enforcement on organized crime by extending the model originally proposed by Garoupa (2000) to multiple criminal organizations, i.e., the Mafias. The role of the Mafias is to regulate the criminal market by extortion. Based on this framework, we can examine the interaction among criminal organizations and the government and see its effect which has not been dealt with in the previous literature. This paper asks the question: Will the multiplicity of the Mafias contribute to the improvement of social welfare?

This paper considers two types of structures of competition among criminal organizations. The first one is criminal organizations face a fierce competition for collecting potential offenders in criminal markets to make profits. The second one is that there exists no such a competition. If criminal organizations suffer from the severe competitions, introducing criminal organizations can not have social welfare improving effects. On the other hand, if there is no competition among them, an oligopolistic criminal markets will contribute to the social welfare improvement. Hence, this paper shows that whether a competition among criminal organizations contributes to social welfare improvement depends on the structure of strategic relations. These results make some differences from the previous literature stressing the desired effects of criminal organizations as in Buchanan (1973), Schelling (1971) and Garoupa (2000). In other words, a monopolistic criminal market controlled by one Mafia does not always preferable.

In addition to a competition for collecting offenders, this paper also considers the possibility of conflicts. By considering the social welfare loss caused in conflicts, the government has to change its optimal strategy. Also, we can obtain new insight as regards to the incentive for engaging in conflicts because

\(^{11}\)The new social welfare function will be \( W = \int_{1/2}^{1} \left( b - h \right) db - (g_1^* + g_2^*)c - cp \) if \( p < p^* \) and \( W = \int_{2/3}^{1} \left( b - h \right) db - cp \) if \( p > p^* \).
it has been unclear whether harsher penalties cause wars. We show that as long as the social welfare loss caused by conflicts is large, the government may have an incentive to take the severe punishment. This is almost consistent with actual policies against organized crime the government takes. As a result, it is not always true that the Mafias contribute to the less severe punishment and the reduction of its expenditure as stressed in the previous literature. As for the incentive for engaging in conflicts, we show that harsher penalties do not always give the Mafias an incentive to wage wars.

This paper’s model has some insufficiencies. First, this paper considers only two different extreme competition situations. However, actual competition among criminal organizations lies between these two extreme case. Hence, more general model should be developed. Second, we should consider the possibility of collusions among the Mafias. In case that the government sets harsh penalties, waging wars is not the only option for the Mafias, on the contrary, they may try cooperative activities against the government.

References


