Subsidies and Self-Funding*

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PRELIMINARY AND INCOMPLETE

Abstract

This paper studies how the combination of subsidies and (implicit or explicit) commitment to self-funding affects R&D and its efficiency. We develop a simple model where a funding body, who does not know the true cost of a project, requires a credit-constrained recipient with private information to commit to a minimum amount of self-funding for the project. We demonstrate that such a commitment can help the funding body to direct funds to recipients who genuinely require subsidies, and mitigates wasteful diversion of funds. Self-funding commitment leads to "crowding-in", whereby the expected private R&D expenditure increases with the amount of subsidies, since higher subsidies induce more high-cost projects to be undertaken whilst ensuring at least some self-funding even for low-cost project. Meanwhile, requiring too much self-funding discourages socially desirable projects due to the recipient's own financing cost. We derive the optimal subsidization policy, which depends crucially on the cost of funds for both the funding body and the recipients, as well as the positive externalities of projects and potential social loss from diversion. We find that i) requiring self-funding commitment may be desirable only if the funding body's financing cost is lower than that of the recipient; and ii) self-funding commitment improves efficiency if the recipient's cost of funding and social loss from diversion are high, and the positive externalities from the project are not too high.

Keywords: Subsidies; Self-funding; Diversion; Matching funds; R&D
JEL Classification: O38; L15

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1 Introduction

Many subsidization schemes require the recipients to bear a fraction of the total cost of a proposed project. This can be implicit, as in cases where applicants are required to present some evidence of quality in the form of a prototype model or pilot study, which may well be costly to develop and often have to be self-funded. A requirement for partial self-funding can also be explicit, as observed in "matching funds" that are used to subsidize projects by private firms, universities, and not-for-profit organizations. For example, matching funds are used in the US by the Department of Defense and the Small Business Administration to subsidize R&D. Government funding for NGOs operating in development countries also tends to take a similar form, such as the ones by the Department for International Development in Britain, the Canadian International Development Agency, and the Swedish International Development Cooperation, to name a few. The National Endowments for the Arts in the US uses the combination of subsidies and commitment to self-financing to provide grants for arts organizations.

When a funding body provides funding to charities, universities, private firms or individuals, there are at least two important economic questions, namely i) whether the proposed project generates enough value (positive externalities) for society; and ii) whether the project genuinely requires public funds to be materialized especially when the applicant has at least some self-funding capability. However, the presence of asymmetric information makes it difficult for funding bodies to provide an appropriate amount of funds, and as a result, applicants may have incentives to request funds for projects even when their externalities relative to the cost are low, or when the projects can actually be materialized without subsidies in order to save directly incurred costs. Excess funds could also lead to, for example, potentially wasteful year-end spending and other diversions of funds. One natural solution for selecting a desirable project and gauging the appropriate amount of subsidy is to conduct intense assessment and monitoring, but recruiting competent and unbiased reviewers may prove difficult especially for projects that involve profit-making opportunities as well as positive externalities. The efficacy of selection may be greatly enhanced if a simple, practical self-selection procedure is entailed in the application process.

This paper studies how subsidies and commitment to self-financing can be used together to select projects that are socially desirable and cannot be realized without the subsidies. We are particularly concerned with subsidies to organizations that may have some self-funding capacity, so that a proposed project might not actually need to be subsidized by the government. Our approach is distinct from, but complementary to the existing literature on R&D subsidies that has primarily focused on the selection and realization of projects with sufficiently large positive externalities. How should commitment to self-financing be used to induce self-selection of projects that are not materialized without subsidies? How does the fraction of the cost to be borne by the recipient organization change according to its budget constraints and the nature of the

proposed project? Many existing matching funds have the matching ratio of 1:1, where a funding body exactly matches the amount of money financed by the recipient. When and how is this ratio socially desirable?

We address these questions by developing a simple screening model that features both the amount of subsidies and the money committed and self-funded by the recipient.

There is a vast literature supporting the rationale of the governmental support on R&D. As Bernstein and Nadiri (1988) shows, the social rate of return of R&D is larger than the private rate. Katz et al. (1990) discusses the difference between social and private incentives on R&D occurs because the profit maximizing firm does not take into account the externality of the innovation. In order to fill the gap, the government gives enough private incentive to develop the new technology by several R&D policies, for example, the intellectual property right (for example, Gallini, 1992; Gilbert and Shapiro, 1990; O'Donoghue et al., 1998; Shapiro, 1985), the prize (for example, Che and Gale, 2003; Fu,Lu and Lu, 2012; Kremer, 1998), and the subsidization. The theoretical literature on the impact of R&D subsidies is scarce in spite of its frequent use as a policy tool.Romano (1989, 1991)consider the R&D policy that is the combination of the patent protection and the subsidies. Romano (1989, 1991)focus on the excess social costs of public funding that is because of the welfare loss from taxation to finance the subsidies and the researcher's moral hazard problem. Romano (1989, 1991) assume that each subsidy has a social cost per dollar. His setting is attractive because of simplicity; however, it is too simple to consider the researcher's incentive to apply the subsidies when his project does not need the governmental support. Kleer (2010) shows the R&D subsidies may be a signal for good project to private investors. In his analysis, the government can screen the projects by setting the quality requirements. In his setting, the researchers do not have enough funds to implement their research. The empirical literature on the impact of R&D mainly focuses on the governmental support causes either crowding out of private R&D or crowding in of it[1]. Theoretically, the subsidies make the unprofitable project to a profitable one by reducing the private cost. Then, the subsidy may stimulate the private spending on R&D. On the other hand, there exists the possibility that the public support crowds out the private expenditure on R&D. The firm has an incentive to use the governmental grants to implement the projects that have been undertaken without the subsidy. The net effect of public support to the private R&D level is not clear from a theoretical point of view.

The rationale of the matching funds is shown by Scotchmer (2004, 2013). She shows that the government can screen out the low value innovation by requiring the researchers to make a commitment of funds to obtain the subsidies.Scotchmer (2013)discusses the subsidies to the university either crowding out the university's R&D expenditure or crowd it in.Scotchmer (2013)shows that the matching funds may

[1]Zúñiga-Vicente et al. (2014)is a good survey about this topic.

mitigate the crowding out problem since the university has to commit to the payment to obtain the governmental support. However, she does not consider the social cost of the matching funds. In this paper, we take into account the social cost of the matching funds and consider when the government should apply the matching funds system.

2 Model

We consider two players, the researcher and the government. The researcher has an idea (m, c), where m means the commercial value of the innovation and c means the cost of developing the idea. In this model, we assume that m is equal to $1/2$ and c is distributed on interval $[0, 1]$. The sequence of the events is:

1. The government designs the matching funds represented by (R, G).
2. The researcher decides whether he will conduct his research by himself, apply the matching funds, or do nothing. If he decides to apply the matching funds, he needs to pay R.
3. The government subsidizes G to the matching fund user who spends all money R for his research[2].

When the researcher conducts his research by himself, his profit is equal to$1/2-(1+\gamma)c$ where $\gamma \geq 0$. In this paper, we assume that the researcher (the government) needs to pay the additional cost $\gamma(\delta)$. $\gamma(\delta)$ includes the funding costs and the opportunity cost of the funding. The researcher's profit when he uses the matching funds depends on the degree of c. If the development cost c is larger than $G + R$, the researcher needs to pay the remaining part. Then, his profit function is given by $1/2-(1+\gamma)R-(1+\gamma)(c-R-G) = 1/2-(1+\gamma)(c-G)$. Otherwise, the researcher will divert the funds for private purposes. We consider the case that there exists the social waste by the diversion. In this case, researcher's profit function is given by $1/2-(1+\gamma)R+(1-\mu)(R+G-c)$. μ is a parameter of social loss by the diversion. If he does not develop his idea, his profit is equal to zero.

The government's utility depends on the positive externality of the innovation and public payment. When the researcher applies the matching funds, the government utility is sum of the researcher's profit and $\alpha/2 - (1 + \delta)G$ where $\alpha \geq 0$ means the externality of the innovation. In this paper, we assume$\alpha$is common knowledge. Only researcher knows the degree of the development cost. The government knows the expected cost. When the researcher conducts his research without government's help, the government's utility is $EU = (1 + \alpha)/2 - (1 + \gamma)E(c)$. When the researcher does not develop his idea, the government utility becomes zero.

[2]The order in which the money does not play an important role in this model. The result does not change if the researcher spends the government subsidies first.

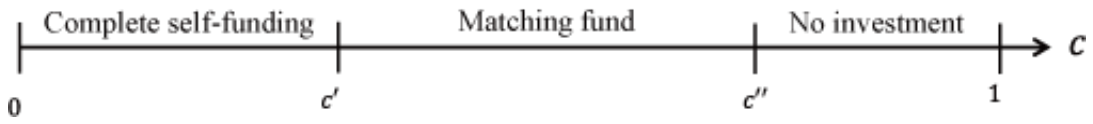


Figure 1: The researcher's action with two cut-off points on c

2.1 Researcher's choice

First, the researcher's behavior is considered in the next proposition.

Proposition 1. *We focus on the cases that there exists the researcher who applies the government subsidy policy. For any $R > 0$ and $G > 0$, the researcher's choice given c is characterized by at most two cutoff points $c' < c''$, where*

if $c \in [0, c']$ he conducts the research purely by self-funding;

if $c \in (c', c'']$ he conducts the research using the matching fund ($R > 0$);

if $c \in (c'', 1]$ he does not conduct the research.

Proof. See the Appendix. □

In this proposition, we ruled out the case that the government sets subsidization policy (R, G) is not used, for example very high R and low G. When we consider the case that $0 < c' < c'' < 1$, there are three types of researcher's action, complete self-funding, matching fund, and no investment as figure 1.

Note that depending on parameter values, c' and/or c'' may not be in the interval $(0, 1)$, in which case the researcher's behavior with respect to c is characterized only by one or two intervals. For example, if $G \geq 1$ and $R = 0$, then we have $c' < 0$ and $c'' > 1$, which implies that the researcher conducts the research using the pure subsidy for any $c \in [0, 1]$, since he does not incur any cost while obtaining both the revenue from the research and any remaining balance $G - c$.

The intuition of this proposition is clear. A researcher's action thus depends on his development cost c, the funding costγ, the parameter of social loss by the diversion μ, and the government's subsidization policy(R, G). When the development cost c is small, the researcher who applies the matching funds needs to pay $(1+\gamma)R-(1-\mu)(R+G-c)$. The first term means the payment to join the matching funds program. The second term means the profit gain by the diversion. If the cost c is relatively small compared to the substantial payment $(R-G)$. When c is large, the researcher needs to pay$(1+\gamma)(c-G)$ under the matching funds. In this case, the researcher can not enjoy the diversion under the matching funds since the development cost is too large.If G is small in this case, no investment is preferable. Otherwise, the researcher will apply the matching fund to invest his idea.This proposition shows the pattern of researcher's choice given the government's policy. The government's strategy is considered in the next section.

Next corollary shows that, under a pure subsidisation scheme $(R = 0, G > 0)$, the researcher's choice with respect to the realized cost c is characterized by at most one cut-off point c'', below which the researcher conducts the research using the subsidy.

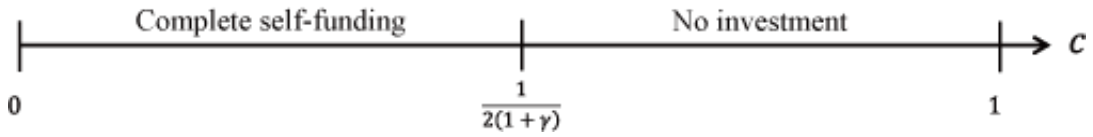


Figure 2: The researcher's behavior without the government subsidization.

Corollary 1. *When we focus on a pure subsidisation case* $(R = 0, G > 0)$*, the researcher's choice given* c *is characterized by at most one cutoff point* c''*, where*

if $c \in [0, c'']$ *he conducts the research using the pure subsidization;*

if $c \in (c'', 1]$ *he does not conduct the research.*

If $c > c''$, the research does not take place as it is too costly for the researcher even with the subsidy. Note that when G is sufficiently high we have $c'' > 1$, which implies that the researcher always conducts the research.

2.2 Government's choice

In this section, we show the optimal government strategies. The government chooses(R, G) to maximize his objective function. We also assume that the government desires to maximize the expected utility gain by using the matching funds. In other words, the government tries to maximize EUG that means differences between the expected utility with the matching funds ($R \geq 0$ and $G > 0$) and that without the government subsidization policy ($R = 0$ and $G = 0$) [3]. We pay attention to the benchmark case that ruled out in Proposition 1.The researcher's choice without the government subsidization is given by figure 2.

In order to consider the optimal government policy, we need to characterize the distribution function of development cost c. However, we can obtain the optimal policy in the limiting case without any assumptions about the cost distribution in the next corollary.

Corollary 2. *(i) When* δ *is sufficiently high, the government will not subsidize at all* $(R = G = 0)$.

(ii) When γ *is sufficiently high, the government will choose either no-subsidization*$(R = G = 0)$ *or full subsidization* $(R = 0, G = 1)$.

The intuition of this corollary is clear. When the government's financial cost δ is large enough, the government stops to subsidize for innovation. When the researcher's funding cost is high, the researcher can not conduct the research by himself. In this case, the government has two options, full subsidization and no-subsidization. If the innovation is socially desirable even if the government gives high enough subsidies, the government will apply the full subsidization. In the middle case, δ and γ are not

[3]This is for expositional convenience. The results in the paper do not change if the government tries to maximize the expected payoff.

so large, the government may prefer the matching fund system. From the government payoff function, we can show the basic trade-off problem under the matching fund. When the government applies the matching fund, he can save the government payment because the researcher whose development cost is low has an incentive to invest by himself instead of applying the matching fund. On the other hand, the matching fund system increases the probability of researcher's self investment, then it increases the funding cost borne by the researcher. This is the negative effect for the government profit since it includes the researcher's profit.

In order to discuss the government's optimal strategy, we need to characterize the distribution function of the development cost c. For simplicity, we assume that c is uniformly distributed between 0 and 1. In this paper, the cases that all projects are implemented by the governmental support are ignored as being impractical. In other words, the externality of the innovation α is not so large.The government strategies are divided to eight regions on (R, G). When the government does not help the researcher's R&D, the expected utility gain is given by 0. The government can set (R, G) that there are two cut-off points on the development cost c. In order to avoid tedious descriptions of equations, we show the expected utility gain in the representative case with matching funds as follows.

$$\begin{aligned} EUG\left(\frac{1}{2(1+\gamma)} \geq R \geq \frac{(1-\mu)G}{\mu+\gamma}\right) = & \left(c'' - \frac{1}{2\,(1+\gamma)}\right) \times \left(\frac{1+\alpha}{2}\right) \\ & + \left(\frac{1}{2\,(1+\gamma)} - c'\right) \times \left(\frac{1}{4} - \frac{(1+\gamma)R}{2}\right) \\ & - \left(R + G - \frac{1}{2\,(1+\gamma)}\right) \times \left((1+\gamma)\,R - \frac{(1-\mu)(1+\gamma)G}{2(\mu+\gamma)}\right) \\ & - (c'' - R - G) \times \left(\frac{1}{4} + \frac{(1+\gamma)R}{2}\right) - (c'' - c') \times (1+\delta)G \end{aligned} \tag{1}$$

Similarly, if the government sets low R and high G, he can eliminate the complete self-funding case. In other words, there are two types of researcher's action, invest with government subsidization and no investment. In this case, the expected payoff gain is

$$\begin{aligned} EUG\,(R=0, G>0) = & \left(c'' - \frac{1}{2\,(1+\gamma)}\right) \times \left(\frac{1+\alpha}{2}\right) \\ & + \left(c'' - \frac{1}{2\,(1+\gamma)}\right) \times \left(\frac{(1-\mu)G}{2} + \frac{1}{4}\right) \\ & - \left(c'' - \frac{1}{2\,(1+\gamma)}\right) \times \frac{1}{4} - c'' \times (1+\delta)G \end{aligned} \tag{2}$$

Equations $(1)$and$(2)$are divided into four parts. The first term of these equations means the utility gain (sum of the commercial value of innovation and the positive externality) from the invested idea because of matching funds. The second term shows the expected researcher's profit gain by switching his decision from the self-investment to the matching funds. The third term means the researcher's expected payment against the research implemented the matching fund program. The last term means the government expected payment for the matching funds. The government sets (R, G) to maximize these functions. Next lemma shows the government strategies.

Lemma 1. *If we assume that the government's strategy is restricted to sub-set there exists the projects are not executed, the government's optimal strategies as follows.*

(i) If there are two cut-off points on c,the expected profit gain is maximized by the choice

$$R^* = \frac{1}{2\,(1+\gamma)}, G^* = \frac{\alpha(\mu+\gamma)}{2(1+\mu+2\delta)(1+\gamma)}.$$

where α must be smaller than $(1+\mu+2\delta)/\,(1-\mu)$ and δ must be larger than γ.

(ii) If all running projects are supported by the subsidization, the expected profit gain is maximized by the choice

$$R^{**} = 0, G^{**} = \frac{\alpha+\alpha\gamma+\gamma-\delta}{2(1+\mu+2\delta)(1+\gamma)}$$

where α must be larger than $(\delta-\gamma)/(1+\gamma)$.

The intuition of this lemma is clear. From proof of proposition 1, we know that the researcher's strategies are divided into eight regions. We ignore three unrealistic regions that are all projects are realized by using assumption. Two of the five regions are corner solutions where the cut-off points c' is equal to zero. In other words, the researcher with zero development cost becomes the indifferent researcher between the self-complete finance and the matching fund with positive (R, G). When the government uses the matching fund system, he can save the payment to the research that can be implemented by the researcher by the screening. In these corner solutions, the government can not receive the benefit at all. Therefore, the government prefers full subsidization policy to these corner solutions. One of three remaining regions is implemented by no subsidization policy. In this setting, the government can increase the expected social welfare by using the appropriate subsidization policies. Therefore, we can narrow down the government strategy given by lemma 1.

The first part of Lemma 1 shows the optimal strategy with the matching funds. The government tries to increases the probability of researcher's self investment by setting positive R. If the researcher can not benefit from the diversion at all $(\mu = 1)$, the government does not subsidize the project not needing the governmental support. When α is larger than the limit $((1+\mu+2\delta)/\,(1-\mu))$, G^*becomes too large to screen out the project should be financed by researcher. If δ is smaller than γ, the government can

not obtain the screening benefit of the matching funds, because the government can raise funds more effectively than the researcher. The second part of this lemma shows the optimal government subsidization. When we assume that the government can only use the full subsidization policy, in other words R must be equal to zero, the optimal government subsidization policy is equal to the second part of Lemma 1. In this case, if α is smaller than $(\delta - \gamma)/(1 + \gamma)$, G^{**}becomes negative. Then, the government can not use the subsidization since we assumed that $G \geq 0$. If the government uses the subsidization, he can obtain the additional utility from the project implemented by the governmental support. On the other hand, the government suffers the funding cost. If α is small, the additional utility from the subsidization is relatively small. Therefore, the government does not have an enough incentive to use that policy. Next propositions shows the government's optimal strategies in this game.

Proposition 2. *(i) When $\delta \leq \gamma$, the government prefers the full subsidization policy.*

(ii) When $\delta > \gamma$,the government applies the matching fund when μ and δ are large and α is not so large.

This proposition shows our main results. As we noticed, if $\delta \leq \gamma$, funding by the researcher is inefficient. In this case, the government can save the social cost by the full subsidization. The matching funds can save the government's funding cost by screen out the project that can be implemented without government's help when $\delta > \gamma$. When μ is large, extra subsidies will become waste for the researcher. In this case, the researcher does not have incentive to apply the matching fund system in order to enjoy the diversion of extra subsidies. In other words, the screening by the matching fund system works well. Then, the government can enjoy the benefit of the matching fund. When α is large, the government will set the higher G because of the social impact of the project. However, large subsidies G will attract the researcher who would like to embezzle the subsidies. If the screening under the matching funds does not work well, the government suffers three types of social cost. First, the researcher tries to benefit from diversion. In this paper, we assume that there exists the social loss by the diversion. Second, the researcher raises funds by inefficient way since δ must be larger than γ when the government applies the matching funds. Third, if the researcher whose development cost is low tries to use the matching funds, he needs to pay relatively large R to join the programme. Then he needs to pay the extra funding cost since it increases in proportion to the payment in this paper. Unless $\mu$is not equal one, the government may prefer the full subsidization policy to the matching fund when the externality of the innovation is large. We shed light on the social cost of the matching funds in this paper. On the other hand, the matching funds can increase the expected government utility when the externality of the innovation is too small to use the subsidization. Next corollary shows the usefulness of the matching funds.

Corollary 3. *When $\delta > \gamma$ and $\alpha < (\delta - \gamma)/(1 + \gamma)$,the government applies the matching fund.*

If $\delta > \gamma$ and$\alpha$is small enough ($\alpha < (\delta - \gamma)/(1 + \gamma)$), the government can not set positive G under the subsidization since the externality of the innovation is small and the government's financing cost is relatively high. The government can increase his expected utility by applying the matching funds even if$\alpha$is too small to set the positive G under the subsidization. When α is small, G^* under the matching funds is also small. Then, the social cost caused by the researcher who desire to enjoy the diversion decreases. Therefore, the matching funds is beneficial for the government even if α is small. In order to understand effect of each policies, next proposition shows the comparative statics of governmental payment G.

Proposition 3. *(i) When the government applies the matching fund, we can obtain the following results.*

$$\frac{\partial G^*}{\partial \mu} > 0, \ \frac{\partial G^*}{\partial \delta} < 0, \ \frac{\partial G^*}{\partial \gamma} \geq 0, \ \frac{\partial G^*}{\partial \alpha} < 0$$

(ii) When the government applies the full subsidization policy, we can obtain the following results.

$$\frac{\partial G^{**}}{\partial \mu} < 0, \ \frac{\partial G^{**}}{\partial \delta} < 0, \ \frac{\partial G^{**}}{\partial \gamma} > 0, \ \frac{\partial G^{**}}{\partial \alpha} < 0$$

If the government funding cost δ becomes large, the government has an incentive to save the payment. On the other hand, if the researcher's funding cost increases, the government increases his payment instead of researcher. When the externality of the innovation α becomes large, the government's incentive to subsidize in order to implement that project. When μ becomes large, the social loss by the researcher's diversion. Therefore, under the full subsidization policy, the government has an incentive to decrease G to prevent this social loss. However, when we consider the matching fund, the government can screen well with large μ. In this case, the government increases G when μincreases since the social benefit by the screening is larger than the social cost of diversion. As we showed the government can screen out the project that the researcher can implement by himself with the matching fund system. If the government can manipulate funds effectively, he may be able to provide more funds to the high development cost than the subsidization.Next proposition shows that when the matching fund system can decrease the number of high cost project more than the subsidization policy.

Proposition 4. *When $\alpha < (\delta - \gamma)/(1 - \mu)$, the matching fund can implement more costly project than the full-subsidization policy.*

In order to understand the intuition of Proposition 4, we need to pay attention to that how the government's expected payoff is affected by an increase of government payment G. When the government increases G, he can obtain the welfare gain by implementation of costly project that is not developed without subsidization. There are three types of social loss by increasing G. First, the government's expected payment

also increases because the probability that the project applies the government help also increases. Second, the social loss by the researcher's diversion also grows by additional G. Third, the social loss caused by the researcher uses the matching fund to fund the project can be implemented without subsidization increases. Third effect does not occurs under the full-subsidization policy. When $\mu$is large andα is not so large, this effect becomes small since the researcher does not have incentive to apply the matching fund and embezzle the extra money. Then, the government under the matching cost can set higher G than that under the full-subsidization policy since the marginal social cost by raising G under the matching fund becomes small.

In reality, the ratio of the researcher contribution in the matching funds is at most 50%. By using this model, we can discuss the optimality of the half and half rule. Next proposition shows that there exists the case that the ratio of the researcher's contribution is larger than 50%.

Proposition 5. *When $\alpha < (1+\mu+2\delta)/(\mu+\gamma)$, the researcher's payment R is larger than the government payment G in the matching funds.*

We can obtain the intuition of this proposition from Proposition 3. The condition given by Proposition 5 is satisfied when μ and γ are not so high and δ is high. These conditions make optimal G under the matching funds large.

3 Proof

Proof of Proposition 1

We consider the optimal researcher's behavior.

1. When $1/2 \geq (1+\gamma)c$, the researcher does not choose not to invest. Then,the researcher will use the matching funds or invest by himself. If $R+G \geq c$, the researcher who applies the matching funds can divert the funds for private purposes. He will invest his idea by himself when $1/2-(1+\gamma)R+(1-\mu) < 1/2-(1+\gamma)c$. By this equation, the researcher will invest by him self when $c < c^*$ where $c^* \equiv R - G(1-\mu)/(\mu+\gamma)$ Otherwise, he will apply the matching funds. If $R+G < c$, the researcher's profit function becomes $1/2-(1+\gamma)(c-G)$ because he needs to pay the remind part. In this case, the researcher will apply the matching fund since it is clear that $1/2-(1+\gamma)(c-G) > 1/2-(1+\gamma)c$.

2. When $1/2 < (1+\gamma)c$ and $1/2(1+\gamma) \geq R$, the researcher does not have an incentive to invest his idea without matching funds. If $R+G > c$, the researcher prefers the matching funds to not to invest his idea if and only if $1/2-(1+\gamma)R+(1-\mu)(R+G-c) \geq 0$. This equation can be rewritten as $c^{**} \geq c$ where $c^{**} \equiv 1/2(1-\mu)-R(\mu+\gamma)/(1-\mu)+G$. If we assume that R is smaller than$1/2(1+\gamma)$, the researcher uses the matching fund since $1/2(1-\mu)-R(\mu+\gamma)/(1-\mu)+G$

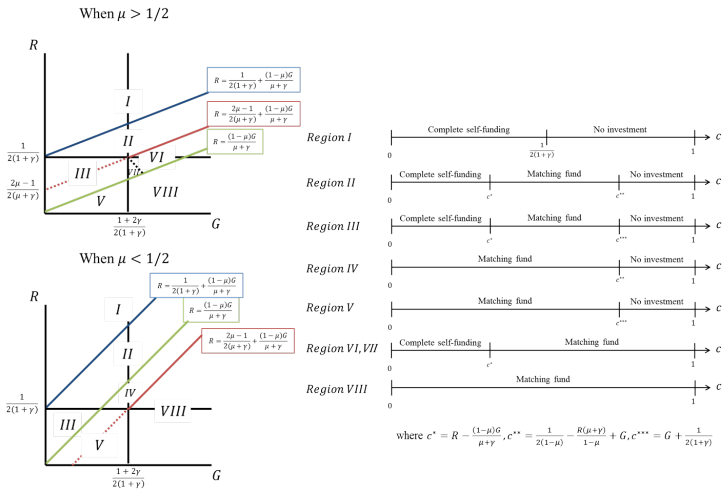


Figure 3: The researcher's strategy

is larger than $R+G$. If $c \geq R+G$, the researcher prefers the matching funds to not to invest his idea if and only if $1/2-(1+\gamma)(c-G) \geq 0$.This equation can be rewritten as $c^{***} \geq c$ where $c^{***} \equiv 1/2(1+\gamma)+G$. If R is smaller than$1/2(1+\gamma)$, $1/2(1+\gamma)+G$ is larger than $R+G$. From these equations, we know that the researcher will apply the matching fund when $1/2(1+\gamma)+G \geq c \geq 1/2(1+\gamma)$. When c is larger than $1/2(1+\gamma)+G$, the researcher will choose not to invest.

3. When $1/2 < (1+\gamma)c$ and $1/2(1+\gamma) < R$, the researcher also does not invest by himself. If $R+G > c$, the researcher prefers the matching funds to not to invest his idea iff $c^{**} \geq c$. If $c \geq R+G$, the researcher prefers does not invest because$c$ is always larger than$1/2(1+\gamma)+G$ when R is larger than $1/2(1+\gamma)$.

From these equations, we can obtain the researcher's strategy. We can show the optimal researcher's action for(R,G) as Figure 3. It is clear that there are one or two cutoff points on the researcher's development cost c.

Proof of Corollary 2

First of all, we need to show the expected government utility function in each regions. The cumulative distribution function of c is given by $F(c)$.

$$EU_{RegionI} = F\left(\frac{1}{2(1+\gamma)}\right) \times \frac{1+\alpha}{2} - F\left(\frac{1}{2(1+\gamma)}\right) \times (1+\gamma)E\left(c \middle| c \leq \frac{1}{2(1+\gamma)}\right)$$

$$
\begin{aligned}
EU_{RegionII} =& F\left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right) \times \frac{1+\alpha}{2} \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \leq c \leq \frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right) \times (1+\delta)G \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma}\right) \times (1+\gamma) E\left(c \middle| c \leq R - \frac{(1-\mu)G}{\mu+\gamma}\right) \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \leq c \leq \frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right) \\
& \quad \times \left((1+\gamma)R - (1-\mu)\left(R + G - E\left(c \middle| R - \frac{(1-\mu)G}{\mu+\gamma} \leq c \leq \frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
EU_{RegionIII} =& F\left(G + \frac{1}{2(1+\gamma)}\right) \times \frac{1+\alpha}{2} - F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \leq c \leq G + \frac{1}{2(1+\gamma)}\right) \times (1+\delta)G \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma}\right) \times (1+\gamma) E\left(c \middle| c \leq R - \frac{(1-\mu)G}{\mu+\gamma}\right) \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \leq c \leq R + G\right) \\
& \quad \times \left((1+\gamma)R - (1-\mu)\left(R + G - E\left(c \middle| R - \frac{(1-\mu)G}{\mu+\gamma} \leq c \leq R + G\right)\right)\right) \\
& -F\left(R + G \leq c \leq G + \frac{1}{2(1+\gamma)}\right) \times (1+\gamma)\left(E\left(c \middle| R + G \leq c \leq G + \frac{1}{2(1+\gamma)}\right) - G\right)
\end{aligned}
$$

$$
\begin{aligned}
EU_{RegionIV} =& F\left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right) \times \frac{1+\alpha}{2} - F\left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right) \times (1+\delta)G \\
& -F\left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right) \\
& \quad \times \left((1+\gamma)R - (1-\mu)\left(R + G - E\left(c \middle| c \leq \frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
EU_{RegionV} =& F\left(G + \frac{1}{2(1+\gamma)}\right) \times \frac{1+\alpha}{2} - F\left(G + \frac{1}{2(1+\gamma)}\right) \times (1+\delta)G \\
& -F(R+G) \times \left((1+\gamma)R - (1-\mu)\left(R + G - E\left(c \middle| c \leq R + G\right)\right)\right) \\
& -F\left(R + G \leq c \leq G + \frac{1}{2(1+\gamma)}\right) \\
& \quad \times (1+\gamma)\left(E\left(c \middle| R + G \leq c \leq G + \frac{1}{2(1+\gamma)}\right) - G\right)
\end{aligned}
$$

$$
\begin{aligned}
EU_{RegionVI} =& \frac{1+\alpha}{2} - F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \le c \le 1\right) \times (1+\delta)G \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma}\right) \times (1+\gamma)E\left(c \middle| c \le R - \frac{(1-\mu)G}{\mu+\gamma}\right) \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \le c \le 1\right) \times \left((1+\gamma)R - (1-\mu)\left(R+G - E\left(c \middle| R - \frac{(1-\mu)G}{\mu+\gamma} \le c \le 1\right)\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
EU_{RegionVII} =& \frac{1+\alpha}{2} - F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \le c \le 1\right) \times (1+\delta)G \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma}\right) \times (1+\gamma)E\left(c \middle| c \le R - \frac{(1-\mu)G}{\mu+\gamma}\right) \\
& -F\left(R - \frac{(1-\mu)G}{\mu+\gamma} \le c \le R+G\right) \\
& \quad \times \left((1+\gamma)R - (1-\mu)\left(R+G - E\left(c \middle| R - \frac{(1-\mu)G}{\mu+\gamma} \le c \le R+G\right)\right)\right) \\
& -F\left(R+G \le c \le 1\right) \times (1+\gamma)\left(E\left(c \middle| R+G \le c \le 1\right) - G\right)
\end{aligned}
$$

$$
\begin{aligned}
EU_{RegionVIII} =& \frac{1+\alpha}{2} - (1+\delta)G \\
& -F(R+G) \times \left((1+\gamma)R - (1-\mu)\left(R+G - E\left(c \middle| 0 \le c \le R+G\right)\right)\right) \\
& -F\left(R+G \le c \le 1\right) \times (1+\gamma)\left(E\left(c \middle| R+G \le c \le 1\right) - G\right)
\end{aligned}
$$

It is clear that when $\delta \to \infty$, the government expected payoff becomes negative except for Region I. When $\gamma \to \infty$,the government expected payoff becomes negative other than Region I and VIII. If the government sets (R, G) to be Regime I, his expected payoff becomes zero since the researcher does not invest his idea . The government has another option to set $R = 0$ and $G = 1$. In this case, the researcher does not have to spend financial cost.

Proof of Lemma 1

To focus on the realistic cases, we ignore the cases that all projects are implemented by the governmental support. In other words, we does not consider the cases α is extremely high (Region VI,VII,VIII). When the government can apply the matching funds, he (or she) can control both $R \ge 0$ and $G \ge 0$ to maximize his expected welfare gain. In order to discuss the government's optimal policy, we need to consider the maximization problems in each regions.

<Region II>

We consider the optimal strategies in Region II. The expected utility gain of the

government in this region is give by the following equation.

$$
\begin{aligned}
max_{R,G}EUG = &\left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G - \frac{1}{2\,(1+\gamma)}\right) \times \left(\frac{1+\alpha}{2}\right) \\
&- \left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G - \frac{1}{2\,(1+\gamma)}\right) \times \left(\frac{(1+\gamma)\,R}{2} + \frac{1}{4} - \frac{(1-\mu)(1+\gamma)G}{2(\mu+\gamma)}\right) \\
&- \left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G - R + \frac{(1-\mu)G}{\mu+\gamma}\right) \times (1+\delta)G
\end{aligned}
$$

From First order condition, we can obtain optimal G^*as follows

$$
G^* = \frac{(\gamma+\mu)\,(\alpha - \alpha\mu - \mu - \delta + 2R\,(\mu+\delta)\,(1+\gamma))}{2(1-\mu)(1+\gamma)(1+\mu+2\delta)}
$$

In this setting, the second derivative of R is positive, then optimalR^*should be equal to the upper bound or lower bound of this region.

When R is equal to the upper bound$(R = 1/2(1+\gamma) + (1-\mu)G/(\mu+\gamma))$,we can obtain

$$
R^* = \frac{\alpha - \alpha\mu + \delta + 1}{2(1+\gamma)(1+\delta)},\ G^* = \frac{\alpha(\mu+\gamma)}{2(1+\delta)(1+\gamma)}. \tag{3}
$$

If the government sets (R, G) as equation (3), any project does not use the government's support. Then, expected profit gain becomes zero.

When R is equal to the lower bound$(R = 1/2(1+\gamma))$,we can obtain

$$
R^* = \frac{1}{2(1+\gamma)},\ G^* = \frac{\alpha(\mu+\gamma)}{2(1+\mu+2\delta)(1+\gamma)}. \tag{4}
$$

If the government sets (R, G) as equation (4), there are two cut-off points on c. In order to guarantee the interior solution,(R^*, G^*)needs to satisfy $R^* - (1-\mu)G^*/(\mu+\gamma) > 0$ and $1/2(1-\mu) - (\mu+\gamma)R^*/(1-\mu) + G^* < 1$. Then,$\alpha$ needs to satisfy following condition.

$$
\alpha < \min\left\{\frac{\mu+2\delta+1}{1-\mu}, \frac{(\mu+2\delta+1)(2\gamma+1)}{\gamma+\mu}\right\}. \tag{5}
$$

In order to focus on the equilibrium that there are three types of project on c, we assume that α satisfies (5).

When $R = (1-\mu)G/(\mu+\gamma)$,we can obtain

$$
R^* = \frac{\alpha - \alpha\mu - \delta - \mu}{2(1+\gamma)(1+\delta)},\ G^* = \frac{(\alpha - \alpha\mu - \delta - \mu)\,(\mu+\gamma)}{2(1+\gamma)(1+\delta)(1-\mu)}. \tag{6}
$$

If the government sets (R, G) as equation (6), there are no project run by complete self funding. When $\mu < 1/2$,(R^*, G^*) will satisfy $1/2(1-\mu) - (\mu+\gamma)R^*/(1-\mu) + G^* < 1$. In addition to that $(1-\mu)G^*/(\mu+\gamma)$ must be larger than $1/2(1+\gamma)$. In order to satisfy this condition. α needs to be larger than $(\mu+2\delta+1)/(1-\mu)$. In our setting, this strategy will

not be chosen. When R becomes lower bound of this region, the expected government payoff becomes positive. Therefore, the government will choose the strategy given by (4) in this region.

<Region III>

We consider the optimal strategies in Region III. The expected utility gain in this region is

$$\begin{aligned} max_{R,G}EUG =& G \times \left(\frac{1+\alpha}{2}\right) + \left(\frac{1}{2\,(1+\gamma)} - R + \frac{(1-\mu)G}{\mu+\gamma}\right) \times \left(\frac{1}{4} - \frac{(1+\gamma)R}{2}\right) \\ & - \left(R + G - \frac{1}{2\,(1+\gamma)}\right) \times \left((1+\gamma)\,R - \frac{(1-\mu)(1+\gamma)G}{2(\mu+\gamma)}\right) \\ & - \left(\frac{1}{2\,(1+\gamma)} - R\right) \times \left(\frac{1}{4} + \frac{(1+\gamma)R}{2}\right) \\ & - \left(G + \frac{1}{2\,(1+\gamma)} - R + \frac{(1-\mu)G}{\mu+\gamma}\right) \times (1+\delta)G. \end{aligned}$$

From First order condition, we can obtain optimal G^*as follows

$$G^* = \frac{(\gamma+\mu)\,(\alpha+\alpha\gamma+\gamma-\delta+2R\,(\delta-\gamma)\,(1+\gamma))}{2(1+\gamma)^2(1+\mu+2\delta)}$$

$R^*$is equal to$1/2(1+\gamma)$ when $\delta > \gamma$ because $\partial E(SWG)/\partial R = G(\delta-\gamma)$.When $\delta > \gamma$,we can obtain

$$R^* = \frac{1}{2(1+\gamma)},\; G^* = \frac{\alpha(\mu+\gamma)}{2(1+\mu+2\delta)(1+\gamma)}. \tag{7}$$

If the government sets (R, G) as equation (7), there are two cut-off points on c. In order to guarantee the interior solution,(R^*, G^*)needs to satisfy $R^* - (1-\mu)G^*/(\mu+\gamma) > 0$ and $G^* + 1/2(1+\gamma) < 1$. Then,α needs to satisfy following condition.

$$\alpha < \min\left\{\frac{\mu+2\delta+1}{1-\mu}, \frac{(\mu+2\delta+1)(2\gamma+1)}{\gamma+\mu}\right\} \tag{8}$$

When $\delta < \gamma$,we can obtain

$$R^* = \frac{(\alpha+\alpha\gamma+\gamma-\delta)\,(1-\mu)}{2(1+\mu+2\gamma)(1+\gamma)(1+\delta)},\; G^* = \frac{(\alpha+\alpha\gamma+\gamma-\delta)\,(\mu+\gamma)}{2(1+\mu+2\gamma)((1+\gamma)(1+\delta)}. \tag{9}$$

When $\delta = \gamma$, ESG does not depend on R. In this case, we can obtain

$$R^* \in \left[\frac{\alpha(1-\mu)}{2(1+\mu+2\gamma)(1+\gamma)}, \frac{1}{2(1+\gamma)}\right],\; G^* = \frac{\alpha(\mu+\gamma)}{2(1+\mu+2\gamma)(1+\gamma)}. \tag{10}$$

<Region IV>

We consider the optimal strategies in Region IV. The utility gain in this region is given by

$$
\begin{aligned}
max_{R,G}EUG = &\left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G - \frac{1}{2\,(1+\gamma)}\right) \times \left(\frac{1+\alpha}{2}\right) \\
&+ \frac{1}{2\,(1+\gamma)} \times \left(\frac{(1-\mu)G - (\gamma+\mu)R}{2}\right) \\
&- \left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G - \frac{1}{2\,(1+\gamma)}\right) \times \left(\frac{1}{4} + \frac{R(\gamma+\mu) - G(1-\mu)}{2}\right) \\
&- \left(\frac{1}{2(1-\mu)} - \frac{R(\mu+\gamma)}{1-\mu} + G\right) \times (1+\delta)G.
\end{aligned}
$$

From First order condition, we can obtain optimal G^* as follows

$$
G^* = \frac{(\alpha - \alpha\mu - \mu - \delta + 2R\,(\mu+\delta)\,(\gamma+\mu))}{2(1-\mu)(1+\mu+2\delta)}
$$

In this setting, the second derivative of R is positive, then optimalR^*should be the upper bound or lower bound of this region. When R is equal to the upper bound$(R = (1-\mu)G/(\mu+\gamma))$,we can obtain

$$
R^* = \frac{\alpha - \alpha\mu - \delta - \mu}{2(\mu+\gamma)(1+\delta)},\ G^* = \frac{\alpha - \alpha\mu - \delta - \mu}{2(1+\delta)(1-\mu)}. \tag{11}
$$

(R^*, G^*)given by (11) satisfies $1/2(1-\mu)-(\mu+\gamma)R^*/(1-\mu)+G^* < 1$ when $\mu < 1/2$.R^*has to be larger than $1/2(1+\gamma)$ in this case. We assume that $R, G \geq 0$, thereforeα needs to be larger than $(\delta+\mu)/(1-\mu)$.Then,α needs to satisfy following condition.

$$
\frac{\gamma + 2\mu + \delta + \gamma\mu + 2\gamma\delta + \mu\delta}{(1+\gamma)(1-\mu)} < \alpha
$$

When R is equal to the lower bound$(R = 1/2(1+\gamma))$,we can obtain

$$
R^* = \frac{1}{2(1+\gamma)},\ G^* = \frac{\alpha + \alpha\gamma - \delta - \mu}{2(1+\mu+2\delta)(1+\gamma)}. \tag{12}
$$

In order to guarantee the interior solution,(R^*, G^*)needs to satisfy $R^* - (1-\mu)G^*/(\mu+\gamma) < 0$ and$1/2(1-\mu) - (\mu+\gamma)R^*/(1-\mu) + G^* < 1$. Then,$\alpha$ needs to satisfy following condition.

$$
\frac{\gamma + 2\mu + \delta + \gamma\mu + 2\gamma\delta + \mu\delta}{(1+\gamma)(1-\mu)} < \alpha < 1 + 2\mu + 3\delta + \frac{\gamma(1+\delta)}{1+\gamma} \tag{13}
$$

When $\alpha$is larger than $(\gamma + 2\mu + \delta + \gamma\mu + 2\gamma\delta + \mu\delta)/(1+\gamma)(1-\mu)$, the expected payoff gain with $R = 1/2(1+\gamma)$ is larger than that with $R = (\alpha - \alpha\mu - \delta - \mu)/2(\mu+\gamma)(1+\delta)$. Therefore, the government will choose (R^*, G^*) given by (12) in this region.

<Region V>

We consider the optimal strategies in Region V. The utility gain in this region is given by

$$max_G EUG = G \times \left(\frac{1+\alpha}{2}\right) + G \times \left(\frac{(1-\mu)G}{2} + \frac{1}{4}\right) - G \times \frac{1}{4} - \left(G + \frac{1}{2\,(1+\gamma)}\right) \times (1+\delta)G$$

From First order condition, we can obtain optimal G^* as follows

$$G^* = \frac{(\alpha + \alpha\gamma + \gamma - \delta + 2R(\gamma+\mu)(1+\gamma))}{2(1+\gamma)(1+\mu+2\delta)}$$

In this setting,the expected payoff gain is a decreasing function of R.Therefore,we can obtain

$$R^* = 0,\ G^* = \frac{\alpha + \alpha\gamma + \gamma - \delta}{2(1+\mu+2\delta)(1+\gamma)}. \tag{14}$$

In order to ignore the cases that the all projects are supported by the government,(R^*, G^*)needs to satisfy $G^*+1/2(1+\gamma) < 1$. In addition to that, we consider the case that $G^*$is positive. Then,α needs to satisfy following condition.

$$\frac{\delta - \gamma}{1+\gamma} \leq \alpha \leq 1 + \mu + 3\delta + \frac{\gamma(\delta+\mu)}{1+\gamma} \tag{15}$$

Now, we are ready to consider the optimal government policy.If we assume that the government's strategy is restricted to sub-set there exists the projects are not executed, it is clear that αneeds to satisfy the following condition from (8), (13) and (15).

$$\alpha \leq \min\left\{1 + \mu + 3\delta + \frac{\gamma(\delta+\mu)}{1+\gamma}, \frac{(\mu+2\delta+1)(2\gamma+1)}{\gamma+\mu}\right\}.$$

We can easily obtain that the utility gain under the optimal strategy in Region IV is smaller than that in Region V when $\alpha$is larger than $(\gamma + 2\mu + \delta + \gamma\mu + 2\gamma\delta + \mu\delta)\,/(1+\gamma)(1-\mu)$. In addition to that, the expected gain under the equation (14) is also larger than that under the strategy given by (9) and(10) when $\delta \leq \gamma$.Therefore, the government optimal strategy is given by equations (4) and (7) because the utility gain under these strategies are positive.Then, we can obtain lemma 1. When the government can not use the matching funds, he (or she) needs to consider the maximization problem with the constraint $R = 0$. In order to discuss the government's optimal policy, we need to consider the maximization problems in each regions. When we assume $R = 0$, the researcher's strategies are restricted to the Region I, V and VIII given by figure 3. We also ignore the cases that all projects are implemented by the governmental support (Region VIII). The maximization problem with the constraint ($R = 0$) in Region V is equal to that showed in proof of lemma 1.

Proof of Proposition 2

From proof of lemma 1, we can calculate the expected government's pay-off gain with each equilibria.

$$EUG\left(R^* = \frac{1}{2\,(1+\gamma)},\ G^* = \frac{\alpha(\mu+\gamma)}{2(1+\mu+2\delta)(1+\gamma)}\right)$$
$$-EUG\left(R^* = 0,\ G^* = \frac{\alpha+\alpha\gamma+\gamma-\delta}{2(1+\mu+2\delta)(1+\gamma)}\right)$$
$$= \frac{(\delta-\gamma)(2\alpha+2\alpha\gamma-\delta+\gamma)-\alpha^2(1+\gamma)(1-\mu)}{8(1+\mu+2\delta)(1+\gamma)^2} \tag{16}$$

The sign of equation (16) depends on the sign of numerator of (16) . If δ is smaller than γ, the sign of (16) must be negative. When $\delta > \gamma$, numerator of (16) may be positive. We can obtain $\alpha^{'}$ that makes equation (16) equal to zero as follows.

$$\alpha^{'} = \frac{(\delta-\gamma)\left(1+\gamma \pm \sqrt{(1+\gamma)(\mu+\gamma)}\right)}{(1-\mu)(1+\gamma)}$$

We know that the government can not set the positive G when α is smaller than$(\delta-\gamma)/(1-\gamma)$. In this case, the government uses the matching fund system to increase his expected utility gain. When $\alpha$is larger than$(\delta-\gamma)/(1-\gamma)$,the government can choose either the matching funds or the full subsidization. It is clear that $(\delta-\gamma)/(1-\gamma)$ is larger than $(\delta-\gamma)\left(1+\gamma-\sqrt{(1+\gamma)(\mu+\gamma)}\right)/(1-\mu)(1+\gamma)$. When$\alpha = (\delta-\gamma)/(1+\gamma)$,(16) becomes positive. In addition to that the second derivative of (16) is negative. Therefore, when α is smaller than$(\delta-\gamma)\left(1+\gamma+\sqrt{(1+\gamma)(\mu+\gamma)}\right)/(1-\mu)(1+\gamma)$,the matching fund system is preferable.

Proof of Proposition 3

From lemma 1, we know that the government sets $G = (\alpha+\alpha\gamma+\gamma-\delta)/2(1+\mu+2\delta)(1+\gamma)$ under the full-subsidization policy. We can calculate as follows.

$$\frac{\partial G}{\partial \mu} = -\frac{\alpha+\alpha\gamma+\gamma-\delta}{2(1+\mu+2\delta)^2(1+\gamma)}$$

$$\frac{\partial G}{\partial \delta} = -\frac{2\alpha+2\alpha\gamma+2\gamma+1+\mu}{(1+\mu+2\delta)^2(1+\gamma)} < 0$$

$$\frac{\partial G}{\partial \gamma} = \frac{1+\delta}{2(1+\mu+2\delta)(1+\gamma)^2} > 0$$

$\partial G/\partial \mu$becomes negative because we assume that $\alpha > (\delta-\gamma)/(1+\gamma)$.

Similarly, we obtain that the government sets $G = \alpha(\mu+\gamma)/2(1+\mu+2\delta)(1+\gamma)$. We also obtain as follows.

$$\frac{\partial G}{\partial \mu} = \frac{\alpha(2\delta-\gamma+1)}{2(1+\mu+2\delta)^2(1+\gamma)}$$

$$\frac{\partial G}{\partial \delta} = -\frac{\alpha(\mu+\gamma)}{(1+\mu+2\delta)^2(1+\gamma)} < 0$$

$$\frac{\partial G}{\partial \gamma} = \frac{\alpha(1-\mu)}{2(1+\mu+2\delta)(1+\gamma)^2} \geq 0$$

$\partial G/\partial \mu$becomes positive because δ must be larger than γ when governments applies the matching fund.

Proof of Proposition 4

In order to capture the effect of matching funds, we consider the government optimal strategy with the constraint $R = 0$.In other words, we discuss the government optimal subsidization. If we assume $R = 0$, the researcher's strategy is described by Region I, V or VII. Since we ruled out Region I and VII by assumptions, the researcher's strategy when$R = 0$ is in Region V.The optimal subsidization G^* in Region V is given by lemma 1 (ii). From lemma 1, we can obtain as follows.

$$G^* - G^{**} = \frac{\delta - \gamma - \alpha(1-\mu)}{2(1+\mu+2\delta)(1+\gamma)}$$

Proof of Proposition 5

From lemma 1, we can obtain as follows.

$$\frac{R^*}{G^*} = 1 + \frac{1+\mu+2\delta-\alpha(\mu+\gamma)}{\alpha(\mu+\gamma)}.$$

From this equation, we know that R^*becomes larger the $G^*$when

$$\alpha < \frac{1+\mu+2\delta}{\mu+\gamma}$$

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