Do Agency Contracts Facilitate Upstream Collusion?*

Yoshifumi Hino[†1,3], Susumu Sato[‡2], and Yusuke Zennyo[§1]

[1]Graduate School of Business Administration, Kobe University

[2]Graduate School of Economics, The University of Tokyo

[3]Business Administration Program, Vietnam–Japan University

This version: July 1, 2019

Abstract

We examine whether agency contracts facilitate collusion among upstream manufacturers, as compared to traditional wholesale contracts. We consider an infinitely repeated game with a monopoly platform and multiple manufacturers. We show that the critical discount factor, above which the upstream collusion can be sustainable by Nash-reversion trigger strategies, is higher under the agency contract than under the wholesale contract. This result indicates that the agency contract does not facilitate upstream collusion in the monopoly platform market. By contrast, in an extended model with competing platforms, we show that the agency contract facilitates upstream collusion. These results provide much implications for several recent antitrust cases on collusion and coordination in online platforms, such as an e-book cartel orchestrated by Apple and a high commission rate imposed by Apple and Google in the mobile application market.

Keywords: *agency contract, cartel stability, upstream collusion, wholesale contract*

JEL Classification: L13, L42, L81, D21, D43, D86, K21

---

*Special appreciation is extended to Jay Pil Choi, Daisuke Hirata, Akifumi Ishihara, Toshihiro Matsumura, Noriaki Matsushima, Hodaka Morita, and Naoki Watanabe. The authors also thank seminar participants at Hitotsubashi University and Kyoto University, and participants at the 2018 annual conference of the JAAE in Kyoto and the 2018 Annual Meeting of the APIOC in Melbourne for their helpful comments.

†E-mail: y.hino@vju.ac.vn

‡E-mail: susumusato@live.jp

§Corresponding author. E-mail: xyzennyo@b.kobe-u.ac.jp

1 Introduction

In March 2016, the U.S. Supreme Court denied a hearing an appeal lodged by Apple Inc., leaving the conclusion that Apple violated *Section 1 of the Sherman Act* by orchestrating a horizontal conspiracy among five of the six largest U.S. book publishers (Hachette, HarperCollins, Macmillan, Penguin, and Simon & Schuster) to raise e-book prices.[1] The U.S. Department of Justice (DOJ) initially filed its complaint that *agency agreements* played an instrumental role.[2] Along with the agency contract, Apple organized a price cartel among the publishers by interacting and sharing information with them. That was a *per se* violation of the Sherman Act. However, it is noteworthy that the district court conceded that the agency agreement is not inherently illegal, and is an entirely lawful contract.

In the case of the e-book cartel, because Apple assembled "a horizontal price-fixing conspiracy" consisting of a group of competitors, it was a *per se* violation of the Sherman Act. Moreover, the district court concluded that even if this case were analyzed under *the rule of reason*, it would still constitute an unreasonable restraint of trade in violation of §1. However, it has not been evaluated whether the agency contract itself facilitates collusion among upstream publishers. This article provides an economic explanation for when and how the agency contract facilitates collusion among publishers.

To this end, we first develop a stylized model of an infinitely repeated game comprising a monopoly platform and multiple manufacturers that sell differentiated products via the platform. The manufacturers can be interpreted as publishers in the example of the e-book market given above. Other examples include e-commerce platform markets (many manufacturers' products are sold on online platforms such as Amazon, eBay, Rakuten in Japan, and Taobao in China) and mobile application markets (application developers distribute their "apps" through Apple App Store and Google Play Store).

We investigate two contract forms: a *wholesale contract* and an *agency contract*, which are the two simplest and most popular contracts adopted by platforms.[3] In the stage-game, for each

[1] See *U.S. v. Apple, Inc.* for more details.

[2] This agency agreement was a contract between a platform and publishers, by which the publishers can set final retail prices. Sales revenue obtained through the platform is split between publishers and the platform subject to a fixed revenue-sharing rule. Recently, such agency contracts have been used widely on other online platforms as well as e-book platforms.

[3] In mobile application markets, agency contracts have been most widely used. By contrast, both the wholesale

contract, the monopoly platform and manufacturers move sequentially explained below. Under the wholesale contract, each manufacturer chooses its wholesale price before the platform sets the retail prices. Under the agency contract, the platform chooses a revenue-sharing rate (a so-called royalty rate or commission rate) before each manufacturer sets the retail price directly. For each contract, we obtain the value of the *critical discount factor*, a minimum discount factor at which joint profit maximizing collusion among manufacturers is sustainable by Nash-reversion trigger strategies. Then, we compare the two critical discount factors to assess whether the agency contract facilitates upstream collusion, or not. We can say that the agency contract facilitates upstream collusion if it reduces the critical discount factor.

Our analysis of the monopoly platform case shows that the critical discount factor is higher under the agency contract than under the wholesale contract, implying the agency contract does not facilitate upstream collusion. The presence of upstream collusion exacerbates the double marginalization problem, which in turn decreases the profit of the platform. Under the agency contract, due to its leadership in the stage-game, the monopoly platform can block the formation of collusion among upstream manufacturers that move later.

In addition, we also examine the extended model with two competing platforms in an effort to explore how the existence of platform competition alters the effect of agency contracts on upstream collusion. Fixing a revenue-sharing rate both the platforms charge, due to revenue-sharing agreements, collusive manufacturers are willing to maximize the total profit across the whole channel, which enables the platforms to obtain the largest revenue-share for the fixed revenue-sharing rate. This, which is in a sharp contrast with the monopoly model, makes deviation less attractive for competing platforms. Therefore, the agency contract facilitates upstream collusion in the markets with competing platforms.

The result obtained implies that introducing agency contracts not only facilitates upstream collusion, but also provokes coordination between competing platforms. In fact, in mobile application markets with competition between Apple App Store and Google Play Store, although developers set the prices of their apps, both Apple and Google impose a 30% commission rate on each purchase.[4]

and agency contracts are widely used, in e-commerce markets. For example, according to Amazon's full-year 2017 financial results, net sales from online stores (i.e., the wholesale model) were $108.4 billion, whereas those from third-party seller services (i.e., the agency model) were $31.9 billion.

[4] In 2016, Apple introduced a new revenue-sharing rule, such that the 30% commission rate will drop to 15% for each subscriber active for at least one year. Google also followed this revenue-sharing rule from the beginning of 2018

That high commission rate led to a key antitrust case that reached the U.S. Supreme Court in November 2018.

The results derived and presented herein represent several important contributions to competition policy. On the one hand, the agency contract itself is not anticompetitive: when the market is served by a monopoly platform, the agency contract does not facilitate upstream collusion. On the other hand, with platform competition, the introduction of agency contracts has an anticompetitive concern to facilitate upstream collusion, and to mitigate platform competition. Therefore, strong revenue-sharing rules should be monitored carefully along with the formation of upstream collusion, which is the second and the most important message of the article.

1.1 Case of e-book cartel

According to *U.S. v. Apple, Inc.*, we summarize the case of the e-book cartel in relation to the purpose of this article.[5]

Amazon, a famous e-book platform, started selling its *Kindle* e-book reader in 2007. First, Amazon signed wholesale contracts with e-book publishers, whereby each publisher sets a wholesale price of its e-book before Amazon charges retail prices for those e-books. In practice, Amazon charged $9.99 for certain new releases and bestselling e-books. When Apple entered the e-book market with its new *iPad* device in 2010, it convinced publishers to adopt agency agreements by which publishers can control the prices of their e-books, with Apple receiving a 30% commission, to break Amazon's monopolistic grip on the market. In addition, Apple included a *most-favored-nation (MFN) clause* in their contract with major publishers, which allowed Apple to sell e-books at its competitors' lowest price (i.e., Amazon's $9.99). The existence of the MFN clause forced publishers to negotiate the signing of an agency contract with Amazon.[6] In consequence, Amazon also moved to the agency contract.

The industrial movement toward agency contracts engendered an increase in e-book retail prices of about 18%, on average. Particularly, the price of *New York Times* bestsellers rose by about 40%. *What caused such an increase in the retail prices of e-books?* On September 5, 2013, the district

in a move to compete better with Apple's offering for iOS developers.

[5]Other examples of digital cartels on online platforms include the case of poster cartels and the Lithuanian Eturas case (Rusu, 2016).

[6]The analyses presented in this article do not incorporate the MFN clause into the model, instead emphasizing how the contract structure affects the sustainability of the upstream price cartel.

court issued a final injunctive order prohibiting Apple from enforcing the MFN clause with e-book publishers and requiring Apple to modify its agency agreements. However, it is noteworthy that the district court conceded that both the agency agreement and MFN clause are not inherently illegal. Therefore, in this case, the increase in e-book retail prices might have resulted from another cause. The district court found evidence indicating that the publishers joined in a horizontal price-fixing conspiracy, undertaken by Apple, to increase the retail price of e-books by eliminating price competition. Apple's central role in the conspiracy was proven as a *per se* violation of the Sherman Act.

1.2 Literature

After an increase in e-book prices associated with Apple's entry into the e-book market in 2010, studies of whether agency agreements raise the retail prices were launched. A seminal work is that of Johnson (2017), who shows that the shift from the wholesale model to the agency model lowers retail prices, which benefits retailers and consumers, but which harms suppliers. Gaudin and White (2014) investigate the bilateral monopoly model with an upstream e-book publisher and a downstream retailer. They show that the wholesale contract engenders a higher retail price if and only if the elasticity of demand strictly decreases as quantity increases. Additionally, they consider an extended case in which the retailer has monopoly market power on an e-book reader device that consumers must purchase to use e-books. In this case, under the wholesale contract, the retailer sets the e-book price at the wholesale price it pays to the publisher and extracts residual surplus through the device price (i.e., two-part tariff pricing). Consequently, no double marginalization problem exists. By contrast, under the agency contract, the double marginalization problem arises because the retailer charges a positive commission fee if the elasticity of demand strictly decreases as quantity increases. As a result, the agency contract can engender a higher retail price.

Foros et al. (2017) consider platform competition with a bilateral duopoly market consisting of two upstream suppliers and two downstream platforms. In their model, under both wholesale and agency contracts, sales revenues are shared between the upstream and the downstream firms. They show that the agency contract engenders higher retail prices than the wholesale contract if and only if the degree of substitution between the platforms is high compared to the degree of substitution between the suppliers. Lu (2017) also analyzes the bilateral duopoly model to demonstrate the procompetitive effect of agency contracts. He shows that the retail prices are lower and the demands are greater under the agency contract than under the wholesale contract, i.e., consumers benefit from the agency contract. The difference from the model of Foros et al. (2017) is the setting of the wholesale contract. He assumes that the upstream firms charge their linear wholesale prices before the downstream firms choose the retail prices, unlike the revenue-sharing contract in Foros et al. (2017). The setting of the stage-game in this article is similar to that of Lu (2017). However, the existing literature on this issue has not considered repeated interactions. This article is the first to examine whether agency contracts facilitate or obstruct upstream collusion.

The present article is also related to the literature on tacit collusion in vertically related markets (e.g., Bernheim and Whinston, 1985; Cooper, 1986; Neilson and Winter, 1993).[7] Recent studies investigate how the sustainability of *upstream* collusion is affected by retail-price maintenance (RPM) agreements (Jullien and Rey, 2007), by vertical mergers (Nocke and White, 2007), and by channel structures (Reisinger and Thomes, 2017). Huang (2017) studies how the sustainability of *downstream* collusion is affected by stationary two-part tariff contracts offered by an upstream supplier before downstream retailers engage in collusive actions infinite-repeatedly. They focus on the traditional wholesale agreements between upstream and downstream firms. That is, unlike this article, none of them considers upstream collusion under the agency contract.

Nevertheless, no study described in the relevant literature addresses the question of whether agency contracts facilitate upstream collusion as compared to traditional wholesale contracts. This study contributes to the literature by showing when and how the agency contract facilitates upstream collusion. The results obtained from this study can present important policy implications.

2 Monopoly Platform

In this section, we examine a model with a monopoly platform and multiple manufacturers. Section 2.1 explains analysis of the stage-game. Section 2.2 presents examination of an infinitely repeated game to derive the critical discount factors under the wholesale and agency contracts; then they are compared to show whether the agency contract facilitates upstream collusion, or not. Detailed proofs are relegated to the Appendix.

[7] The framework used by Bernheim and Whinston (1985) and Cooper (1986) is a finite horizon; it is not what we mean by collusion in our manuscript.

2.1 The stage-game

We describe the setting of the stage-game; then we derive the static equilibrium and the collusive equilibrium, respectively, under the wholesale and agency contracts. Lastly, those stage-game outcomes are compared.

Setting of the stage-game We consider a vertically related market with a monopoly platform P and n manufacturers ($n \geq 2$). Each manufacturer i ($i = 1, \cdots, n$) produces good i and sells it through platform P. Let p_i and q_i be the price and the quantities of good i sold on the platform. The utility function of a representative consumer is given as

$$u(q_1, \cdots, q_n) = \sum_i aq_i - \sum_i \frac{b}{2}q_i^2 - \sum_{i \neq j} b\theta q_i q_j - \sum_i p_i q_i \ , \tag{1}$$

which yields the inverse demand function as $p_i = a - bq_i - b\theta \sum_{j \neq i} q_j$. Solving those inverse demand functions with respect to quantity yields the demand function of good i as

$$q_i = \alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \ , \tag{2}$$

where $\alpha \equiv \frac{a}{b\{1+(n-1)\theta\}}$, $\beta \equiv \frac{\{1+(n-2)\theta\}}{b(1-\theta)\{1+(n-1)\theta\}}$, and $\gamma \equiv \frac{\theta}{b(1-\theta)\{1+(n-1)\theta\}}$. We let $\sigma \equiv \beta - (n-1)\gamma > 0$, which means that the own-price effect dominates the sum of cross-price effects.

This article presents examination of two contract forms: wholesale and agency. The stage-game of each contract is presented as follows. Under the wholesale contract, each manufacturer i sets its wholesale price w_i in the first round before platform P charges a pair of retail prices $(p_1, \cdots, p_n)$ in the second round. We assume that manufacturers incur a constant marginal cost of production $c > 0$. The profits of manufacturer i and of platform P are given respectively as $\pi_i = (w_i - c)q_i$ and $\Pi = \sum_i (p_i - w_i)q_i$. In contrast, under the agency contract, the platform offers a revenue-sharing rule $s \in [0, 1]$ for all manufacturers in the first round before each manufacturer i sets its retail price p_i in the second round. The profits of manufacturer i and of platform P are given respectively as $\pi_i = \{(1-s)p_i - c\}q_i$ and $\Pi = \sum_i sp_iq_i$.

In the symmetric equilibrium (i.e., $p_i = p$ and $q_i = q$ for all i), consumer surplus is given as $CS = nq^2/(2\sigma)$. Additionally, we interpret social welfare as a sum of consumer surplus and profits

Table 1: Stage-game equilibrium and collusive equilibrium under the wholesale contract

$k = \{N, C\}$	Stage-game Nash equilibrium (N)	Collusive equilibrium (C)
w^*_{Wk}	$\frac{\alpha+\beta c}{\beta+\sigma}$	$\frac{\alpha}{2\sigma} + \frac{c}{2}$
p^*_{Wk}	$\frac{\alpha}{2\sigma} + \frac{\alpha+\beta c}{2(\beta+\sigma)}$	$\frac{3\alpha+\sigma c}{4\sigma}$
q^*_{Wk}	$\frac{\beta(\alpha-\sigma c)}{2(\beta+\sigma)}$	$\frac{\alpha-\sigma c}{4}$
π^*_{Wk}	$\frac{\beta(\alpha-\sigma c)^2}{2(\beta+\sigma)^2}$	$\frac{(\alpha-\sigma c)^2}{8\sigma}$
Π^*_{Wk}	$n \cdot \frac{\beta^2(\alpha-\sigma c)^2}{4\sigma(\beta+\sigma)^2}$	$n \cdot \frac{(\alpha-\sigma c)^2}{16\sigma}$
CS^*_{Wk}	$n \cdot \frac{\beta^2(\alpha-\sigma c)^2}{8\sigma(\beta+\sigma)^2}$	$n \cdot \frac{(\alpha-\sigma c)^2}{32\sigma}$
SW^*_{Wk}	$n \cdot \frac{\beta(3\beta+4\sigma)(\alpha-\sigma c)^2}{8\sigma(\beta+\sigma)^2}$	$n \cdot \frac{7(\alpha-\sigma c)^2}{32\sigma}$

of all firms, i.e., $SW = CS + \Pi + \sum_i \pi_i$.

Analysis of the stage-game under the wholesale contract One can consider the situation in which the game above is played only once under the wholesale contract. The stage-game is solved using backward induction. In the second round, given a pair of wholesale prices $w = (w_1, \cdots, w_n)$, platform P chooses retail prices $(p_1, \cdots, p_n)$ to maximize its profit. Solving $\partial\Pi/\partial p_i = 0$ for all i implies that $p_i(w_i) = \alpha/(2\sigma) + w_i/2$.

Then, the maximization problem faced by manufacturer i in the first round is written as

$$\max_{w_i} \ \pi_i(w) = (w_i - c)\left(\alpha - \beta p_i(w_i) + \gamma \sum_{j \neq i} p_j(w_j)\right). \tag{3}$$

Solving $\partial\pi_i/\partial w_i = 0$ for all i implies that $w^*_{WN} = (\alpha + \beta c)/(\beta + \sigma)$. We here use subscript W to indicate the *wholesale* contract, subscript N to denote the stage-game *Nash* equilibrium, and superscript '$*$' to represent the equilibrium result of the monopoly platform model. The equilibrium retail price, profit of manufacturers, profit of the platform, consumer surplus, and social welfare are presented in Table 1.

Next, we derive the collusive stage-game equilibrium. The best response of platform P at the second round is unchanged, i.e., $p_i(w_i) = \alpha/(2\sigma) + w_i/2$. In the first round, manufacturers collude in their wholesale prices to maximize their joint profit, which is given as $\pi^C = \sum_i \pi_i = \sum_i (w_i - c)q_i$.

The maximization problem of cartel party (i.e. manufacturers) is the following.

$$\max_{w_1,\cdots,w_n} \pi^C(w) = \sum_i (w_i - c)\left(\alpha - \beta p_i(w_i) + \gamma \sum_{j\neq i} p_j(w_j)\right) \tag{4}$$

Solving this problem yields $w^*_{WC} = \alpha/(2\sigma) + c/2$. We use subscript C to denote the *collusive* equilibrium. Other outcomes are presented in Table 1.

By comparing the stage-game Nash equilibrium with the collusive result obtained under the wholesale contract, we derive several results indicating how upstream collusion affects the vertical relation.

Lemma 1. *Consider the case with a monopoly platform. Under the wholesale contract, upstream collusion increases the wholesale and retail prices (i.e.,* $w^*_{WN} < w^*_{WC}$ *and* $p^*_{WN} < p^*_{WC}$*), which decreases the quantities demanded, consumer surplus, and social welfare (i.e.,* $q^*_{WN} > q^*_{WC}$*,* $CS^*_{WN} > CS^*_{WC}$*, and* $SW^*_{WN} > SW^*_{WC}$*). By colluding, the manufacturers receive greater profit (i.e.,* $\pi^*_{WN} < \pi^*_{WC}$*), although the profit of the platform declines (i.e.,* $\Pi^*_{WN} > \Pi^*_{WC}$*).*

Under the wholesale contract, collusion among manufacturers increases the wholesale price, which in turn induces the platform to choose higher retail prices. The wholesale price increased by the collusion raises the manufacturers' margin per unit sold, which increases their profits. Simultaneously, the resulting higher retail price shrinks the quantities demanded; it thereby decreases the profit of the platform, consumer surplus, and social welfare.

Analysis of the stage-game under the agency contract As in the case of the wholesale contract, we consider a situation in which the stage-game is played only once under the agency contract. In the second round, each manufacturer i chooses its retail price p_i independently, given the revenue-sharing rule s chosen by the platform. Solving $\partial\pi_i/\partial p_i = 0$ for all i implies $p_{AN}(s) = \left(\alpha + \beta\frac{c}{1-s}\right)/(\beta+\sigma)$, where we use subscript A to denote the *agency* contract. The corresponding quantities, profit of manufacturers, and profit of platform P are given respectively as

$$q_{AN}(s) = \frac{\beta\left(\alpha - \sigma\frac{c}{1-s}\right)}{\beta+\sigma}, \tag{5}$$

$$\pi_{AN}(s) = \beta(1-s)\left(\frac{\alpha - \sigma\frac{c}{1-s}}{\beta+\sigma}\right)^2, \tag{6}$$

$$\Pi_{AN}(s) = \sum_i \left(s \cdot \frac{\alpha + \beta \frac{c}{1-s}}{\beta+\sigma} \cdot \frac{\beta\left(\alpha - \sigma\frac{c}{1-s}\right)}{\beta+\sigma} \right). \tag{7}$$

In the first round, the platform sets the revenue-sharing rule s^*_{AN} which satisfies the first-order condition, which is equivalent to

$$\begin{aligned}
\frac{\partial \Pi_{AN}(s^*_{AN})}{\partial s} &= \frac{n\beta}{(\beta+\sigma)^2} \cdot \left[\begin{array}{l} \left(\alpha + \beta\frac{c}{1-s^*_{AN}}\right)\left(\alpha - \sigma\frac{c}{1-s^*_{AN}}\right) \\ + \frac{s^*_{AN}c}{(1-s^*_{AN})^2}\left\{\alpha(\beta-\sigma) - 2\beta\sigma\frac{c}{1-s^*_{AN}}\right\} \end{array} \right] = 0, \\
&\iff \frac{n\beta}{(\beta+\sigma)^2} \cdot \frac{1}{(1-s^*_{AN})^3} \cdot \xi(s^*_{AN}) = 0,
\end{aligned} \tag{8}$$

where $\xi(s) \equiv \alpha^2(1-s)^3 + \left\{(\beta-\sigma)\alpha c + \beta\sigma c^2\right\}(1-s) - 2\beta\sigma c^2$. s^*_{AN} is unique over $s \in (0, 1-\sigma c/\alpha)$ and satisfies the second-order condition.[8] The equilibrium outcomes of the stage game are written with s^*_{AN}, that is, $p^*_{AN} = p_{AN}(s^*_{AN})$, $q^*_{AN} = q_{AN}(s^*_{AN})$, $\pi^*_{AN} = \pi_{AN}(s^*_{AN})$, and $\Pi^*_{AN} = \Pi_{AN}(s^*_{AN})$, which are presented in Table 2.[9]

Next, we derive the collusive stage-game equilibrium. Consider collusion among manufacturers in the second round, where the manufacturers seek to maximize their joint profit for any given revenue-sharing rule s set by the platform. Let $\pi^C = \sum_i \pi_i = \sum_i \{(1-s)p_i - c\}q_i$. Solving $\partial\pi^C/\partial p_i = 0$ for all i implies that $p_{AC}(s) = \left(\alpha + \sigma\frac{c}{1-s}\right)/(2\sigma)$. The corresponding quantities, profit of manufacturers, and profit of platform P respectively denote given as shown below.

$$q_{AC}(s) = \frac{\alpha - \sigma\frac{c}{1-s}}{2}, \tag{9}$$

$$\pi_{AC}(s) = (1-s)\frac{\left(\alpha - \sigma\frac{c}{1-s}\right)^2}{4\sigma} \tag{10}$$

---

[8] It holds that sign $\frac{\partial \Pi_{AN}(s)}{\partial s}$ = sign $\xi(s)$ for all $s \in (0,1)$. We have

$$\begin{aligned}
\xi(0) =& (\alpha + \beta c)(\alpha - \sigma c) > 0, \\
\xi\left(1 - \frac{\sigma c}{\alpha}\right) =& -\frac{c^2\sigma}{\alpha}(\sigma+\beta)(\alpha - \sigma c) < 0, \\
\xi'(s) =& -3\alpha^2(1-s)^2 - \left\{(\beta-\sigma)\alpha c + \beta\sigma c^2\right\} < 0.
\end{aligned}$$

The first two inequalities ensure the existence of the solution in $(0, 1-\sigma c/\alpha)$. The last inequality ensures uniqueness of the solution which satisfies the second-order condition.

[9] The fact that $s_{AN} \in (0, 1-\sigma c/\alpha)$, which we have already shown in footnote 8, ensures all outcomes to be positive (i.e., $p_{AN} > 0$, $q_{AN} > 0$, $\pi_{AN} > 0$, and $\Pi_{AN} > 0$).

Table 2: Stage-game equilibrium and collusive equilibrium under the agency contract

$k=\{N,C\}$	Stage-game Nash equilibrium (N)	Collusive equilibrium (C)
s^*_{Ak}	s^*_{AN} in equation (8)	s^*_{AC} in equation (12)
p^*_{Ak}	$\frac{1}{\beta+\sigma}\left(\alpha+\beta\frac{c}{1-s^*_{AN}}\right)$	$\frac{1}{2\sigma}\left(\alpha+\sigma\frac{c}{1-s^*_{AC}}\right)$
q^*_{Ak}	$\frac{\beta}{\beta+\sigma}\left(\alpha-\sigma\frac{c}{1-s^*_{AN}}\right)$	$\frac{1}{2}\left(\alpha-\sigma\frac{c}{1-s^*_{AC}}\right)$
π^*_{Ak}	$\frac{\beta(1-s^*_{AN})}{(\beta+\sigma)^2}\left(\alpha-\sigma\frac{c}{1-s^*_{AN}}\right)^2$	$\frac{1-s^*_{AC}}{4\sigma}\left(\alpha-\sigma\frac{c}{1-s^*_{AC}}\right)^2$
Π^*_{Ak}	$n\cdot\frac{\beta s^*_{AN}}{(\beta+\sigma)^2}\left(\alpha+\beta\frac{c}{1-s^*_{AN}}\right)\left(\alpha-\sigma\frac{c}{1-s^*_{AN}}\right)$	$n\cdot\frac{s^*_{AC}}{4\sigma}\left(\alpha^2-\frac{\sigma^2c^2}{(1-s^*_{AC})^2}\right)$
CS^*_{Ak}	$\frac{n\beta^2}{2\sigma(\beta+\sigma)^2}\left(\alpha-\sigma\frac{c}{1-s^*_{AN}}\right)^2$	$\frac{n}{8\sigma}\left(\alpha-\sigma\frac{c}{1-s^*_{AC}}\right)^2$
SW^*_{Ak}	$CS^*_{AN}+\Pi^*_{AN}+n\cdot\pi^*_{AN}$	$CS^*_{AC}+\Pi^*_{AC}+n\cdot\pi^*_{AC}$

$$\Pi_{AC}(s)=\sum_i\left(s\cdot\frac{\alpha+\sigma\frac{c}{1-s}}{2\sigma}\cdot\frac{\alpha-\sigma\frac{c}{1-s}}{2}\right) \tag{11}$$

In the first round, the platform sets the revenue-sharing rule s^*_{AC} which satisfies the following first-order condition.[10]

$$\begin{aligned}\frac{\partial\Pi_{AC}(s^*_{AC})}{\partial s}&=\frac{n}{4\sigma}\cdot\left[\left(\alpha^2-\frac{\sigma^2c^2}{(1-s^*_{AC})^2}\right)-s^*_{AC}\frac{2\sigma^2c^2}{(1-s^*_{AC})^3}\right]=0\\ \iff&\ \frac{n}{4\sigma}\cdot\left\{\alpha^2-\frac{(1+s^*_{AC})\sigma^2c^2}{(1-s^*_{AC})^3}\right\}=0\end{aligned} \tag{12}$$

The collusive outcomes of the stage game are written with s^*_{AC}, i.e., $p^*_{AC}=p_{AC}(s^*_{AC})$, $q^*_{AC}=q_{AC}(s^*_{AC})$, $\pi^*_{AC}=\pi_{AC}(s^*_{AC})$, and $\Pi^*_{AC}=\Pi_{AC}(s^*_{AC})$, which are presented in Table 2.[11]

By comparing the stage-game Nash equilibrium with the collusive one under the agency contract, we derive the following lemma.

[10] s^*_{AC} is unique over $(0,1-\sigma c/\alpha)$ and satisfies the second-order condition. We obtain

$$\begin{aligned}\frac{\partial\Pi^C_A}{\partial s}(0)&=\frac{n}{4\sigma}(\alpha+\sigma c)(\alpha-\sigma c)>0,\\ \frac{\partial\Pi^C_A}{\partial s}\left(1-\frac{\sigma c}{\alpha}\right)&=\frac{2n\alpha^2}{\sigma c}(\alpha-\sigma c)<0,\\ \frac{\partial^2\Pi^C_A}{\partial s^2}(s)&<0,\quad\forall s\in\left(0,1-\frac{\sigma c}{\alpha}\right).\end{aligned}$$

Therefore, s^*_{AC} is uniquely decided over $(0,1-\sigma c/\alpha)$ and satisfies the second-order condition.

[11] The fact that $s^*_{AC}\in(0,1-\sigma c/\alpha)$, which we have already shown in footnote 10, ensures that all outcomes are positive (i.e., $p^*_{AC}>0$, $q^*_{AC}>0$, $\pi^*_{AC}>0$, and $\Pi^*_{AC}>0$).

Lemma 2. *Consider the case with a monopoly platform. Under the agency contract, upstream collusion decreases the revenue-sharing rule (i.e., $s^*_{AN} > s^*_{AC}$), but it increases the retail price (i.e., $p^*_{AN} < p^*_{AC}$), which engenders lower demand, consumer surplus, and social welfare (i.e., $q^*_{AN} > q^*_{AC}$, $CS^*_{AN} > CS^*_{AC}$, and $SW^*_{AN} > SW^*_{AC}$). By colluding, the manufacturers receive greater profit (i.e., $\pi^*_{AN} < \pi^*_{AC}$), whereas the profit of the platform is lower in the presence of the manufacturers' collusion (i.e., $\Pi^*_{AN} > \Pi^*_{AC}$).*

The monopoly platform's optimal revenue-sharing rule is lower in the presence of the manufacturers' collusion. This is because upstream collusion exacerbates the double marginalization problem, making it unattractive for the platform to impose a high revenue-sharing rule. Lemma 2 also shows that, even though upstream collusion induces the platform to set the lower revenue-sharing rule, it results in the higher retail price eventually.

Upstream collusion enables manufacturers to win the favorable revenue-share and to charge the higher retail price. Therefore, they can gain the greater profit in the collusive equilibrium. By contrast, the platform loses its profit and therefore has no incentive to foster the collusion.

Finally, as in the wholesale contract, upstream collusion shrinks consumer demand. It therefore degrades consumer surplus and social welfare.

Comparison of collusive outcomes under the two contracts Comparing the collusive outcome under the wholesale contract with the one under the agency contract, we have the following lemma.

Lemma 3. *Consider the case with a monopoly platform. Presume also that the upstream manufacturers collude to maximize their joint profit. Compared to the wholesale contract, the retail price is lower under the agency contract (i.e., $p^*_{WC} > p^*_{AC}$), which engenders greater demand (i.e., $q^*_{WC} < q^*_{AC}$). Then, consumer surplus and social welfare are higher under the agency contract (i.e., $CS^*_{WC} < CS^*_{AC}$ and $SW^*_{WC} < SW^*_{AC}$). The manufacturers receive greater profit under the wholesale contract (i.e., $\pi^*_{WC} > \pi^*_{AC}$), although the platforms prefer the agency contract (i.e., $\Pi^*_{WC} < \Pi^*_{AC}$).*

When the manufacturers collude to maximize their joint profit, the retail price is lower under the agency contract than under the wholesale contract, which generates the greater demand and then improves both consumer surplus and social welfare. This is because revenue-sharing under the agency contract mitigates the double-marginalization problem. In this respect, the agency contract is not necessarily anticompetitive, but rather improves social welfare.

Furthermore, the manufacturers prefer the wholesale contract to the agency contract, whereas the platform has the opposite preference. In other words, every firm prefers a contract that guarantees itself the first move. This result is also presented by Johnson (2017), although upstream collusion is not considered in his model. Consequently, the presence of upstream collusion does not alter the preference of each firm related to the contract type.

2.2 Infinitely repeated game

This subsection presents examination of an infinitely repeated game in which the monopoly platform and manufacturers play the above stage-game over period ($t = 1, 2, \cdots, \infty$). The game is of common knowledge and perfect monitoring. Let $\delta \in (0, 1)$ be a common discount factor. The payoff of each player is given as the sum of the discounted stage-game payoff. They maximize their expected payoff.

We assume that manufacturers sustain their joint profit maximizing collusion through infinite Nash-reversion, where any deviation by an upstream manufacturer is followed by the infinite repeated play of the subgame-perfect equilibrium of the stage game. We determine the value of the critical discount factor, which is the lowest discount factor with which manufacturers' joint profit maximizing collusion can be sustained.[12] We stipulate that the agency contract *facilitates* upstream collusion if it *reduces* the critical discount factor.

Critical discount factor under the wholesale contract We use δ_W^* to denote the critical discount factor under the wholesale contract. Manufacturers collude to maximize their joint profit before the platform moves. This is the standard case of the literature on collusion in the vertically related market.

We have already derived the stage-game Nash equilibrium and the collusive equilibrium in Section 2.1. Consequently, here, we compute the deviation payoff. The best response of platform

[12] Generally, many studies in the literature on Repeated Games are interested in the derivation of the set of the equilibrium payoffs for a *fixed* discount factor. In contrast, we follow the recent Industrial Organization literature on collusion in vertical related markets (e.g., Nocke and White, 2007; Reisinger and Thomes, 2017) that are interested in the derivation of the value of the critical discount factor.

P at the second round is $p_i(w_i) = \alpha/(2\sigma) + w_i/2$. We consider the deviation by manufacturer 1 in the first round, with no loss of generality. Consequently, the other manufacturers set the collusive wholesale price (i.e., $w_j = w^*_{WC}$ for $j = 2, \cdots, n$). Then, the profit of manufacturer 1 can be written as

$$\pi_1^D(w_1) = (w_1 - c)\left(\alpha - \beta p_1 + \gamma \sum_{j \neq 1} p_j\right) = (w_1 - c)\left\{\alpha - \beta\left(\frac{\alpha}{2\sigma} + \frac{w_1}{2}\right) + (\beta - \sigma)\left(\frac{\alpha}{2\sigma} + \frac{w^*_{WC}}{2}\right)\right\}. \tag{13}$$

Solving $\partial \pi_1^D(w_1)/\partial w_1 = 0$ yields $w^*_{WD} = \{\alpha(\beta + \sigma) + \sigma(3\beta - \sigma)c\}/(4\beta\sigma)$. We use subscript D to denote the *deviation* outcome. The deviation profit can be computed as $\pi^*_{WD} = (\beta + \sigma)^2(\alpha - \sigma c)^2/(32\beta\sigma^2)$.

Then collusion among manufacturers is sustainable if and only if the following inequality holds.

$$\frac{\pi^*_{WC}}{1-\delta} \geq \pi^*_{WD} + \frac{\delta \pi^*_{WN}}{1-\delta} \tag{14}$$

Solving condition (14) with equality determines the critical discount factor under the wholesale contract, δ^*_W.

Proposition 1. *Consider the case with a monopoly platform. Under the wholesale contract, collusion among manufacturers is sustainable if and only if the discount factor is sufficiently high to satisfy*

$$\delta \geq \delta^*_W \equiv \frac{\beta^2 + \sigma^2 + 2\beta\sigma}{\beta^2 + \sigma^2 + 6\beta\sigma}. \tag{15}$$

One can readily show that the critical discount factor δ^*_W increases with n and γ and that it decreases in β, i.e., $\partial \delta^*_W/\partial n > 0$, $\partial \delta^*_W/\partial \gamma > 0$, and $\partial \delta^*_W/\partial \beta < 0$. The tougher competition (e.g., a larger number of manufacturers and/or a higher degree of substitution) engenders the higher δ^*_W, which is relevant to a well-known result that the collusion becomes more difficult to sustain as the competition gets tougher.[13]

Critical discount factor under the agency contract Next, we derive the critical discount factor under the agency contract, denoted as δ^*_A. Unlike the wholesale contract, under the agency

[13]The opposite result is obtained in the common retailer case of Reisinger and Thomes (2017). That is, they demonstrated that the critical discount factor decreases in γ when each manufacturer offers a two-part tariff contract to the common retailer.

contract, one player moves before the formation of collusion. In other words, the monopoly platform can design its action strategically to influence collusion among manufacturers. The platform might have an incentive to hinder or foster the collusion by manipulating its action. Thus, we need to consider the equilibrium path by which the platform deters manufacturers' joint profit maximization.

Because this situation largely differs from the standard of the literature, let us formally describe all players' strategies. We consider two states: State C (collusion) and State P (punishment). The initial period begins with State C. Players' strategies for each state are described as follows.

State C: Monopoly platform chooses a revenue-sharing rule s. All manufacturers take a symmetric action that maximizes their joint profit for the given s, that is, $p_i = p_{AC}(s)$ for all i. If a different revenue-sharing rate s' is chosen by the platform, manufacturers play the Nash equilibrium strategy, i.e., $p_i = p_{AN}(s')$ for all i. The state turns to State P if any manufacturer sets $p_i \neq p_{AC}(s)$ even as the platform chooses s.

State P: The subgame perfect equilibrium of the stage-game is played. The state of the next period is State P.

We assume that, if manufacturers observe a different revenue-sharing rate in a period, they stop colluding tentatively in that period and play the Nash equilibrium strategy. In other words, the monopoly platform is allowed to temporarily deter the formation of upstream collusion by charging a different revenue-sharing rate. Note that, even if the platform chooses a different revenue-sharing rate, the state remains to be State C.

Because Lemma 2 shows $\Pi^*_{AN} > \Pi^*_{AC}$, the platform has an incentive to set a different revenue-sharing rule s' $(= s_N)$ in an effort to hinder the collusion in every period.

Proposition 2. *Consider the case with a monopoly platform. Under the agency contract, the platform deters the formation of upstream collusion in every period. Therefore, as compared to the wholesale contract, the agency contract does not facilitate upstream collusion (i.e., $\delta^*_W < \delta^*_A = 1$).*

Basically, the presence of upstream collusion exacerbates the double marginalization problem, which in turn decreases the profit of the platform. By definition, under the agency contract, the platform can temporarily block the formation of upstream collusion simply by charging a different revenue-sharing rule. On the equilibrium path, in every period, the monopoly platform deters cartel formation because of the potential danger of upstream collusion. Thus, $\delta_A^* = 1$ holds.

As one would expect, this result largely depends on the definition of strategy. However, the main result $\delta_W^* < \delta_A^*$, which implies that the agency contract does not facilitate upstream collusion in the monopoly platform market, can be robust even under more relaxed definitions of strategy, which is demonstrated in Section 4.1.

3 Platform Competition

In the previous section, we show that the agency contract does not facilitate upstream collusion in the monopoly platform market. The purpose of this section is to address how the presence of platform competition affects the sustainability of upstream collusion.

3.1 The stage-game

Herein, after we describe only those components of the stage-game that differ from those of the monopoly platform case, we analyze static equilibrium and collusion among manufacturers under the two contracts. Subsequently, we compare those stage-game outcomes.

Setting of the stage-game We consider two upstream manufacturers ($u = 1, 2$) and two downstream platforms ($d = 1, 2$). Both manufacturers produce differentiated goods with a constant marginal cost, which is normalized to zero. Let p_d^u and q_d^u be the price and the quantities of manufacturer u's good demanded at platform d.

Following Dobson and Waterson (1996), we assume a representative consumer with the following utility function:[14]

$$U = \sum_{u,d} q_d^u - \sum_{u,d} \frac{1}{2}(q_d^u)^2 - \lambda(q_d^u q_{-d}^u + q_d^{-u} q_{-d}^{-u}) - \mu(q_d^u q_d^{-u} + q_{-d}^u q_{-d}^{-u}) - \lambda\mu(q_d^u q_{-d}^{-u} + q_{-d}^u q_d^{-u}) \quad \text{for } u, d = 1, 2, \tag{16}$$

where $\mu \in (0,1)$ and $\lambda \in (0,1)$ respectively represent the degrees of substitution between the upstream manufacturers and between the downstream platforms.

[14] This utility function and the resulting demand function have been used widely in the literature (e.g., Johansen and Vergé, 2017; Foros et al., 2017; Lu, 2017).

Table 3: Stage-game equilibrium and collusive equilibrium under the wholesale contract with platform competition

$k = \{N, C\}$	Stage-game Nash equilibrium (N)	Collusive equilibrium (C)
w^{**}_{Wk}	$\frac{1-\mu}{2-\mu}$	$\frac{1}{2}$
p^{**}_{Wk}	$\frac{(1-\lambda)(2-\mu)+1-\mu}{(2-\lambda)(2-\mu)}$	$\frac{3-2\lambda}{2(2-\lambda)}$
q^{**}_{Wk}	$\frac{1}{(1+\lambda)(1+\mu)(2-\lambda)(2-\mu)}$	$\frac{1}{2(1+\lambda)(1+\mu)(2-\lambda)}$
π^{**}_{Wk}	$\frac{2(1-\mu)}{(1+\lambda)(1+\mu)(2-\lambda)(2-\mu)^2}$	$\frac{1}{2(1+\lambda)(1+\mu)(2-\lambda)}$
Π^{**}_{Wk}	$\frac{2(1-\lambda)}{(1+\lambda)(1+\mu)(2-\lambda)^2(2-\mu)^2}$	$\frac{1-\lambda}{2(1+\lambda)(1+\mu)(2-\lambda)^2}$
CS^{**}_{Wk}	$\frac{2}{(1+\lambda)(1+\mu)(2-\lambda)^2(2-\mu)^2}$	$\frac{1}{2(1+\lambda)(1+\mu)(2-\lambda)^2}$
SW^{**}_{Wk}	$\frac{2(7-4\lambda-4\mu+2\lambda\mu)}{(1+\lambda)(1+\mu)(2-\lambda)^2(2-\mu)^2}$	$\frac{7-4\lambda}{2(1+\lambda)(1+\mu)(2-\lambda)^2}$

Utility-maximization subject to the budget constraint yields the inverse demand functions: $p^u_d = 1 - (q^u_d + \lambda q^u_{-d}) - \mu(q^{-u}_d + \lambda q^{-u}_{-d})$ for $u, d = 1, 2$. Solving for quantities, we can derive the demand function as follows.

$$q^u_d = \frac{(1-\lambda)(1-\mu) - p^u_d + \mu p^{-u}_d + \lambda(p^u_{-d} - \mu p^{-u}_{-d})}{(1-\lambda^2)(1-\mu^2)} \quad \text{for} \quad u, d = 1, 2 \tag{17}$$

Analysis of the stage-game under the wholesale contract Under the wholesale contract, manufacturer u sets wholesale prices (w^u_1, w^u_2) in the first round; then platform d sets retail prices (p^1_d, p^2_d) in the second round.

First, we derive the stage-game Nash equilibrium. In the second round, solving the platforms' profit maximization problems, we obtain the second-stage price $p^u_d(w)$. Using this, the corresponding second-round quantities are given as

$$q^u_d(w) = \frac{\lambda(w^u_d - \mu w^{-u}_d) - (2-\lambda^2)(w^u_d - \mu w^{-u}_d)}{(4-\lambda^2)(1-\lambda^2)(1-\mu^2)} + \frac{1}{(2-\lambda)(1+\lambda)(1+\mu)}. \tag{18}$$

In the first round, the stage-game equilibrium can be derived by solving the following problem for each manufacturer: $\max_{(w^u_d)_{d=1,2}} \sum_{d'} (w^u_{d'} - c) q^u_{d'}(w)$, as analyzed by Lu (2017). The outcomes of the stage-game Nash equilibrium are presented in Table 3. We use superscript '$**$' to denote the equilibrium in the model with the two competing platforms.

Second, we consider joint profit maximization by manufacturers, which can be derived by solving the following problem: $\max_{(w_d^u)_{u,d=1,2}} \sum_{u'} \sum_{d'} (w_{d'}^{u'} - c) q_{d'}^{u'}(w)$. The resulting outcomes of the collusive equilibrium are also presented in Table 3.

By comparing the stage-game Nash equilibrium with the collusive one under the wholesale contract, we derive the following lemma.

Lemma 4. *Consider a market with two competing platforms. Under the wholesale contract, upstream collusion increases wholesale and retail prices (i.e., $w_{WN}^{**} < w_{WC}^{**}$ and $p_{WN}^{**} < p_{WC}^{**}$), which decrease the quantities demanded, consumer surplus, and social welfare (i.e., $q_{WN}^{**} > q_{WC}^{**}$, $CS_{WN}^{**} > CS_{WC}^{**}$, and $SW_{WN}^{**} > SW_{WC}^{**}$). The manufacturers receive greater profit (i.e., $\pi_{WN}^{**} < \pi_{WC}^{**}$), whereas the profit of platforms declines (i.e., $\Pi_{WN}^{**} > \Pi_{WC}^{**}$).*

Lemma 4 presents the same result as that of Lemma 1. Therefore, under the wholesale contract, the effects of upstream collusion are irrelevant to the existence of platform competition. As in the monopoly platform case, the upstream collusion is profitable only to the manufacturers; it is deleterious to the platforms, consumer surplus, and social welfare.

Analysis of the stage-game under the agency contract Under the agency contract, platform d sets a revenue-sharing rule s_d in the first round; then manufacturer u sets retail prices (p_1^u, p_2^u) in the second round.

We first derive the stage-game Nash equilibrium. In the second round, given (s_1, s_2), each manufacturer u solves the following problem:

$$\max_{p_1^u,\ p_2^u} \ (1 - s_1) p_1^u q_1^u + (1 - s_2) p_2^u q_2^u. \tag{19}$$

Solving the maximization problem above, we derive that $p_d^1 = p_d^2$ holds for $d = 1, 2$, implying that competing manufacturers charge the same retail price for each platform. Then, we use $p_d^N(s_d, s_{-d})$ and $q_d^N(s_d, s_{-d})$ to represent the resulting price and the corresponding quantities for platform d given (s_d, s_{-d}), respectively.

In the first round, each platform chooses revenue-sharing rule s_d to maximize $\Pi_d^N(s_d, s_{-d}) = 2 s_d p_d^N(s_d, s_{-d}) q_d^N(s_d, s_{-d})$. By solving this problem, we obtain a symmetric Nash equilibrium s_{AN}^{**}. This stage-game Nash equilibrium has been analyzed by Foros et al. (2017) and Lu (2017), as presented in Table 4.

Table 4: Stage-game equilibrium and collusive equilibrium under the agency contract with platform competition

$k = \{N, C\}$	Stage-game Nash equilibrium (N)	Collusive equilibrium (C)
s^{**}_{Ak}	$\frac{(2-\mu)(1-\lambda^2)}{2-\mu-\lambda\mu}$	$1-\lambda^2$
p^{**}_{Ak}	$\frac{1-\mu}{2-\mu}$	$\frac{1}{2}$
q^{**}_{Ak}	$\frac{1}{(1+\lambda)(1+\mu)(2-\mu)}$	$\frac{1}{2(1+\lambda)(1+\mu)}$
π^{**}_{Ak}	$\frac{2\lambda(1-\mu)(2\lambda-\mu-\lambda\mu)}{(1+\lambda)(1+\mu)(2-\mu)^2(2-\mu-\lambda\mu)}$	$\frac{\lambda^2}{2(1+\lambda)(1+\mu)}$
Π^{**}_{Ak}	$\frac{2(1-\lambda)(1-\mu)}{(1+\mu)(2-\mu)(2-\mu-\lambda\mu)}$	$\frac{1-\lambda}{2(1+\mu)}$
CS^{**}_{Ak}	$\frac{2}{(1+\lambda)(1+\mu)(2-\mu)^2}$	$\frac{1}{2(1+\lambda)(1+\mu)}$
SW^{**}_{Ak}	$\frac{6-4\mu}{(1+\lambda)(1+\mu)(2-\mu)^2}$	$\frac{3}{2(1+\lambda)(1+\mu)}$

Second, we consider joint profit maximization by manufacturers. In this case, given (s_1, s_2), each manufacturer u solves the following problem:

$$\max_{(p^u_d)_{u,d=1,2}} \sum_{u=1,2} \{(1-s_1)p^u_1 q^u_1 + (1-s_2)p^u_2 q^u_2\}. \tag{20}$$

Solving the maximization problem above, we derive the retail prices, which are denoted as $p^C_d(s_d, s_{-d})$ for $d = 1, 2$. Let $q^C_d(s_d, s_{-d})$ be the resulting quantities.

In the first round, platform d solves the maximization problem: $\max_{s_d} \sum_{u=1,2} s_d p^C_d(s_d, s_{-d}) q^C_d(s_d, s_{-d})$. The first-order condition is given as

$$\sum_{u=1,2} \left(p^C_d q^C_d + s_d \left\{ \frac{\partial p^C_d}{\partial s_d} \left(q^C_d + \frac{\partial q^C_d}{\partial p_d} p^C_d \right) + \frac{\partial p^C_{-d}}{\partial s_d} \frac{\partial q^C_d}{\partial p_{-d}} p^C_d \right\} \right) = 0. \tag{21}$$

Invoking symmetry, $s_d = s_{-d} = s$, we obtain $s^{**}_{AC} = 1 - \lambda^2 > s^{**}_{AN}$. The other resulting outcomes are also presented in Table 4.

By comparing the stage-game Nash equilibrium with the collusive one under the agency contract, we derive the following lemma.

Lemma 5. *Consider the case with two competing platforms. Under the agency contract, the revenue-sharing rule and the retail price are higher in the presence of the manufacturers' collusion (i.e., $s^{**}_{AN} > s^{**}_{AC}$ and $p^{**}_{AN} < p^{**}_{AC}$), which decrease the quantities demanded, consumer surplus, and* *social welfare (i.e., $q_{AN}^{**} > q_{AC}^{**}$, $CS_{AN}^{**} > CS_{AC}^{**}$, and $SW_{AN}^{**} > SW_{AC}^{**}$). Both the manufacturers and the platforms receive greater profits (i.e., $\pi_{AN}^{**} < \pi_{AC}^{**}$ and $\Pi_{AN}^{**} > \Pi_{AC}^{**}$).*

Lemma 5 presents the same result as that of Lemma 2. Therefore, also under the agency contract, the effects of upstream collusion are irrelevant to the existence of platform competition. The upstream collusion is profitable only to the manufacturers; it is deleterious to the platforms, consumer surplus, and social welfare.

Comparison of collusive outcomes under the two contracts Here, we compare the collusive outcome under the wholesale contract with the outcome obtained under the agency contract.

Lemma 6. *Suppose that two platforms compete in the market. Presume that the upstream manufacturers collude to maximize their joint profit. Compared to the wholesale contract, the retail price is lower under the agency contract (i.e., $p_{WC}^{**} > p_{AC}^{**}$), which engenders greater demand, consumer surplus, and social welfare (i.e., $q_{WC}^{**} < q_{AC}^{**}$, $CS_{WC}^{**} < CS_{AC}^{**}$, and $SW_{WC}^{**} < SW_{AC}^{**}$). The manufacturers receive greater profit under the wholesale contract (i.e., $\pi_{WC}^{**} > \pi_{AC}^{**}$), although the platforms prefer the agency contract (i.e., $\Pi_{WC}^{**} < \Pi_{AC}^{**}$).*

Lemma 6 shows qualitatively the same result as that obtained in Lemma 3. Therefore, when manufacturers collude to maximize their joint profit, the existence of platform competition does not affect the comparison between the two contracts.

3.2 Infinitely repeated game

In this subsection, we consider an infinitely repeated game in which two platforms and two manufacturers play the above stage-game over period ($t = 1, 2, \cdots, \infty$) in order to derive the values of the critical discount factor for the respective contracts

Critical discount factor under the wholesale contract We use δ_W^{**} to denote the critical discount factor under the wholesale contract. As in the monopoly case, we consider the standard Nash-reversion trigger strategies in the literature on collusion in the vertically related market.

We have already derived the stage-game Nash equilibrium and the collusive equilibrium in Section 3.1. Therefore, we compute the deviation payoff here. The platforms' best response strategies given a pair of wholesale prices are unchanged if a manufacturer deviates. Then, the first-order condition for the best deviation from the joint profit maximization can be written as

$$\frac{\partial \pi^u}{\partial w_d^u} = q_d^u + w_d^u \frac{\partial q_d^u}{\partial w_d^u} + w_{-d}^u \frac{\partial q_{-d}^u}{\partial w_d^u} = 0 \quad \text{s.t. } w_d^{-u} = w^C \quad \text{for } d = 1, 2. \tag{22}$$

Solving this equation, we obtain the deviation strategy $w_{WD}^{**} = (2-\mu)/4$ and the corresponding profit of the deviating manufacturer $\pi_{WD}^{**} = (2-\mu)^2/\{8(2-\lambda)(1+\lambda)(1-\mu^2)\}$.

Consequently, the critical discount factor δ_W^{**} is computed as follows.

$$\delta_W^{**} = \frac{\pi_{WD}^{**} - \pi_{WC}^{**}}{\pi_{WD}^{**} - \pi_{WN}^{**}} = \frac{(2-\mu)^2}{8 - 8\mu + \mu^2} \tag{23}$$

Proposition 3. *In the model with two competing platforms, collusion under the wholesale contract among manufacturers is sustainable if and only if* $\delta \geq \delta_W^{**}$.

Critical discount factor under the agency contract Next, we derive the critical discount factor under the agency contract, denoted as δ_A^{**}. In order to confine our attention to symmetric equilibrium, we consider the following Nash-reversion trigger strategies.

State C: Platforms choose the same revenue-sharing rule, $(s_1, s_2) = (s, s)$. All manufacturers take a symmetric action that maximizes their joint profit for the given s. If a platform d sets a different revenue-sharing rate s', manufacturers play the Nash equilibrium strategy for $(s_d, s_{-d}) = (s', s)$. The state turns to State P if any manufacturer takes a different action even under $(s_1, s_2) = (s, s)$.

State P: The subgame perfect equilibrium of the stage-game is played. The state of the next period is State P.

The definition of strategy implies that, as in the monopoly platform case, platforms can temporarily deter the formation of upstream collusion by charging a different revenue-sharing rule. In what follows, we will show that the agency contract facilitate upstream collusion even with such the definition of strategy that seems to make the cartel formation difficult.

We have already derived the stage-game Nash equilibrium in Section 3.1. In addition, to derive the critical discount factor, we need to consider the manufacturers' collusive strategy given

$(s_1, s_2) = (s, s)$, and the optimal deviation from that collusive situation.

First, let us take a closer look at the manufacturers' collusive pricing given $(s_1, s_2) = (s, s)$. The first-order condition for joint profit maximization given $(s_1, s_2) = (s, s)$ is written by

$$\frac{\partial(\pi^1 + \pi^2)}{\partial p^u_{dA}} = (1 - s)\left(q^u_d + p^u_{dA}\frac{\partial q^u_d}{\partial p^u_d} + p^{-u}_d\frac{\partial q^{-u}_d}{\partial p^u_d} + p^u_{-dA}\frac{\partial q^u_{-d}}{\partial p^u_d} + p^{-u}_d\frac{\partial q^{-u}_{-d}}{\partial p^u_d}\right) = 0. \tag{24}$$

Under the symmetric revenue-sharing rate, all retail prices are the same (i.e., $p^1_1 = p^2_1 = p^1_2 = p^2_2$). Thus, we use $p^C(s, s)$ to denote the resulting retail price. The resulting price and the corresponding quantities, profit of manufacturers, and profit of platforms are given as follows.

$$p^C(s, s) = \frac{1}{2} \tag{25}$$

$$q^C(s, s) = \frac{1}{2(1 + \lambda)(1 + \mu)} \tag{26}$$

$$\pi^C(s, s) = \frac{1 - s}{2(1 + \lambda)(1 + \mu)} \tag{27}$$

$$\Pi^C(s, s) = \frac{s}{2(1 + \lambda)(1 + \mu)} \tag{28}$$

Next, we look at the optimal deviation from the above collusive pricing. Given $(s_1, s_2) = (s, s)$, the optimal deviation from joint profit maximization is obtained by solving the following problem.

$$\max_{p^u_1, p^u_2} \quad (1 - s)(p^u_1 q^u_1 + p^u_2 q^u_2) \quad s.t. \quad p_1^{-u} = p_2^{-u} = p^C(s, s) \tag{29}$$

Solving the above maximization problem, we can show that the best deviation is charging $p^D(s, s) = (2 - \mu)/4$. The resulting profit for the deviating manufacturer is given by $\pi^D(s, s) = (1 - s)(2 - \mu)^2/\{8(1 + \lambda)(1 - \mu^2)\}$.

Now, let us consider the manufacturers' incentive for deviation given $(s_1, s_2) = (s, s)$. Their joint profit maximization is sustainable if and only if the following inequality holds:

$$\frac{\pi^C(s, s)}{1 - \delta} \geq \pi^D(s, s) + \frac{\delta \pi^{**}_{AN}}{1 - \delta} \tag{30}$$

$$\Longleftrightarrow \delta \geq \frac{(1 - s)(2 - \mu)^2\mu^2(2 - \mu - \lambda\mu)}{(1 - s)(2 - \mu)^4(2 - \mu - \lambda\mu) - 16\lambda(1 - \mu)^2(2\lambda - \mu - \lambda\mu)} \equiv \delta_A(s) \tag{31}$$

The critical discount factor given s, $\delta_A(s)$, has the following property.

Lemma 7. *The following statements hold: (i)* $\delta'_A(s) > 0$ *and (ii)* $\delta_A(s_N^{**}) = \delta_W^{**}$.

Lemma 7 (i) implies that the upstream collusion becomes increasingly difficult to sustain as the platforms set a higher revenue-sharing rule and (ii) shows that, when the platforms set $s_1 = s_2 = s_N^{**}$, the critical discount factor becomes equal to the critical discount factor under the wholesale contract.

Finally, we consider the platforms' incentives for charging a different revenue-sharing rate in order to temporarily block the cartel formation. Although Lemma 5 shows $\Pi_{AN}^{**} > \Pi_{AC}^{**}$, for a fixed s, the following lemma holds.

Lemma 8. $\Pi_d^N(s,s) < \Pi^C(s,s)$ *for any* $s \in (0,1)$.

This lemma states that the platforms are better off with upstream collusion for any fixed s, which would discourage the competing platforms from taking unexpected actions. As a result, the presence of platform competition can radically alter the effect of agency contracts on collusion among manufacturers: The agency contract facilitates upstream collusion, which is summarized in the following proposition.

Proposition 4. *When two platforms compete in a market, agency contracts facilitate upstream collusion. Formally,* $\delta_A^{**} < \delta_W^{**}$ *is satisfied.*

The detailed proof is delegated to the Appendix. Some intuition can be provided for why the presence of platform competition drastically alters the attitude of platforms towards upstream collusion.

When upstream manufacturers collude, they seek to concentrate their sales into a platform with lower s. Thus, competing platforms have a stronger incentive to set a lower s than their rival. That is, upstream collusion makes platform competition fiercer, resulting in lower platform profits, as shown in Lemma 5. In contrast, fixing $(s_1, s_2) = (s, s)$, the presence of upstream collusion heightens profits of competing platforms, as shown in Lemma 8, which is a sharp contrast to the monopoly platform case. The reason is the following. Given $(s_1, s_2) = (s, s)$, due to revenue-sharing agreements, the objective that collusive manufacturers maximize is equivalent to the total profit across the whole channel. Thus, upstream collusion enables the platforms to obtain the largest revenue-share for the fixed revenue-sharing rate, which makes it less attractive for them to deviate from $(s_1, s_2) = (s, s)$.

Interestingly, repeated interactions provoke not only collusion among the cartel party (i.e., manufacturers), but also coordination among the non-cartel party (i.e., platforms). In this respect, the result obtained might be related to the fact that Apple actively organized an e-book price cartel among five of the six largest U.S. book publishers. Additionally, the result could also be helpful to understand the high commission rate of 30% imposed by Apple and Google in the mobile application market. In fact, this attention-getting antitrust case reached the U.S. Supreme Court.

4 Discussion

In this section, we discuss several issues missing from the main analysis.

4.1 On the deviation by monopoly platform

In Proposition 2, we show that the agency contract does not facilitate upstream collusion in the monopoly platform market. However, players' strategies considered are strict about cartel formation among upstream manufacturers: Any deviation by monopoly platform temporarily breaks down upstream collusion. One might expect that the result of Proposition 2 crucially relies on this strong assumption. The purpose of this subsection is to confirm the robustness of the result even under more relaxed definition about players' strategies.

Before describing each player's strategy, let us define a set $\mathcal{S}(x)$ as follows.

Definition 1. *$\mathcal{S}(x)$ is a set of revenue-sharing rates at which manufacturers sustain their joint profit maximization given that monopoly platform will continue to charge $s = x$ from the next period onwards.*

In this subsection, using $\mathcal{S}(x)$ defined above, we weaken the extent to which a deviation by monopoly platform blocks the formation of upstream collusion. In particular, we assume the following manufacturers' response to deviations by the platform. Suppose that monopoly platform deviates from charging s and chooses s' in a period. If s' lies within $\mathcal{S}(x)$, then upstream manufacturers sustain their joint profit maximizing collusion in that period. Otherwise, if $s' \notin \mathcal{S}(x)$, manufacturers play the Nash equilibrium strategy for the given s' at that period. Note that our analysis in Section 2.2 is corresponding to the situation in which $\mathcal{S}(x)$ is defined as $\mathcal{S}(x) = \{x\}$.

In sum, we consider the following strategy for each player.

State C: Monopoly platform chooses a revenue-sharing rule s. All manufacturers take a symmetric action that maximizes their joint profit as long as the revenue-sharing rule set by the platform lies within $\mathcal{S}(s)$. If the platform sets a different revenue-sharing rate $s' \notin \mathcal{S}(s)$, then manufacturers play the Nash equilibrium strategy for the given s'. The state turns to State P if any manufacturers do not choose the collusive action even as the platform chooses a revenue-sharing rate that lies within $\mathcal{S}(s)$.

State P: The subgame perfect equilibrium of the stage-game is played. The state of the next period is State P.

Moreover, we focus on the *platform-preferred* revenue-sharing rate s^* that maximizes the profit of the monopoly platform under the strategies stated above. Formally, the *platform-preferred* revenue-sharing rate can be described as follows.

$$s^* = \arg\max_s \Pi_A(s, \mathcal{S}(s)), \tag{32}$$

where

$$\Pi_A(s, \mathcal{S}(s)) = \begin{cases} \Pi_{AC}(s) & \text{if } s \in \mathcal{S}(s) \\ \Pi_{AN}(s) & \text{if } s \notin \mathcal{S}(s) \end{cases} \tag{33}$$

We investigate whether the upstream collusion can be sustainable or not under the *platform-preferred* revenue-sharing rate, which depends on the value of discount factor δ. Therefore, we explore the minimum value of discount factors at which the *platform-preferred* revenue-sharing rate is determined so as to make the upstream collusion sustainable.

Proposition 5. *The platform-preferred revenue-sharing rule s^* is given by*

$$s^* = \begin{cases} s^*_{AN} & \text{if } \delta < \delta^*_A \ , \\ s^*_{AC} & \text{if } \delta \geq \delta^*_A \ , \end{cases} \tag{34}$$

where $\delta^*_A = \delta^*_W$.

The proof is delegated to Appendix B. This proposition shows that the critical discount factor under the agency contract is the same as that under the wholesale contract, which confirms the robustness of Proposition 2 that the agency contract does not facilitate upstream collusion.

4.2 Observability of actions

For the main analysis, we have assumed perfect monitoring, i.e., manufacturers can observe all actions of the others. In reality, however, there might be situations in which wholesale prices set by other manufacturers are not publicly observable, although the retail prices are easier to observe. It is well-known in the literature of collusion under imperfect monitoring that the more difficult it is to observe each player's action, the more difficult it is to sustain the collusion.[15] Consequently, under the wholesale contract, one would have the higher critical discount factor if the wholesale prices were not publicly observable. That fact enhances the robustness of our results such that the agency contract might be more likely to facilitate collusion among the manufacturers.

5 Conclusion

This article addresses the important issue of the sustainability of a digital cartel. To address the relation between cartel sustainability and the contract form, we develop a stylized model of the infinitely repeated game. Then we obtain and compare the critical discount factors, a minimum discount factor at which the price cartel among upstream manufacturers can be sustainable, under a wholesale contract and an agency contract.

The central message from our study is that the agency contract does not facilitate upstream collusion in the case of monopoly platform, although it facilitates upstream collusion with the platform competition. Our results are expected to contribute to the literature by providing important policy implications. The agency contract is not necessarily *per se illegal*, but competition authorities must be more concerned about it when several platforms compete in the market.

From the other viewpoint, however, given upstream collusion to be sustained, revenue-sharing agreements under agency contracts mitigate the double marginalization problem, which generates higher consumer surplus and social welfare compared to the wholesale contract. In this regard, the agency contract can not necessarily be characterized as anticompetitive even in markets with platform competition.

[15] See Jullien and Rey (2007) for this argument applied to retail-price maintenance.

We conclude by describing the limitations of our model and by discussing potential avenues for future research. First, we do not examine the effect of Most-Favored-Nation (MFN) clauses. In the real case of the e-book cartel, however, the clause played a central role in the proceedings along with the agency agreements.[16] Secondly, we assume that, in the stage-game of the wholesale contract, manufacturers move before the platform(s). In the e-book market before Apple's entry, Amazon had set retail prices of most e-books at $9.99 and had committed to such a pricing policy. It might be valuable to analyze the wholesale contract with realistic timing of moves, where platforms move before manufacturers.[17] Lastly, in the model of platform competition, we compare the sustainability of collusion between two symmetric situations where both platforms impose the same contract (wholesale or agency) on their manufacturers. As typically observed in Amazon's behavior in the e-book market, however, platforms can choose their contract forms strategically. In this regard, it would be interesting to consider the platforms' endogenous choices about which contract to select, and to confirm whether and when the agency contract is adopted in equilibrium. These extensions require more complex analysis, which is beyond the scope of this article.

Appendix A: Proofs

Proof of Lemma 2.

First, we show $s^*_{AN} > s^*_{AC}$. We focus on the first-order conditions with respect to s, i.e., equations (8) and (12). Consider that the revenue-sharing rule is set at the one in the collusive case (i.e., $s = s^*_{AC}$). The derivative of platform's profit under the punishment phase is computed by

$$\frac{\partial \Pi_{AN}(s^*_{AC})}{\partial s} = \frac{n\beta}{(\beta+\sigma)^2} \cdot \left[\begin{array}{l} \left(\alpha + \beta \frac{c}{1-s^*_{AC}}\right)\left(\alpha - \sigma \frac{c}{1-s^*_{AC}}\right) \\ + \frac{s^*_{AC} c}{(1-s^*_{AC})^2} \left\{ \alpha(n-1)\gamma - 2\beta\sigma \frac{c}{1-s^*_{AC}} \right\} \end{array} \right] \tag{A.1}$$

$$= \frac{n\beta}{(\beta+\sigma)^2} \cdot \left[\alpha^2 + \frac{\alpha(\beta-\sigma)c}{(1-s^*_{AC})^2} - \frac{(1+s^*_{AC})\beta\sigma c^2}{(1-s^*_{AC})^3} \right] \tag{A.2}$$

[16] For example, Boik and Corts (2016), Johansen and Vergé (2017), Johnson (2017), and Maruyama and Zennyo (2018) are noteworthy for providing analyses of so-called a Most-Favored-Customer clause or a price parity clause.

[17] Timing of moves of this type has been analyzed in Marketing Science and Operations Research, and so-called *retailer-stackelberg* game (e.g., Choi, 1991).

$$= \frac{n\beta}{(\beta+\sigma)^2} \cdot \left[\frac{(1+s^*_{AC})\sigma^2 c^2}{(1-s^*_{AC})^3} + \frac{\alpha(\beta-\sigma)c}{(1-s^*_{AC})^2} - \frac{(1+s^*_{AC})\beta\sigma c^2}{(1-s^*_{AC})^3} \right] \tag{A.3}$$

$$= \frac{n\beta}{(\beta+\sigma)^2} \cdot \frac{(\beta-\sigma)c}{(1-s^*_{AC})^3} \cdot \left\{ (1-s^*_{AC})\alpha - (1+s^*_{AC})\sigma c \right\}, \tag{A.4}$$

where the third equality follows from equation (12).

If $\partial\Pi_{AN}(s^*_{AC})/\partial s > 0$, then $s^*_{AN} > s^*_{AC}$. Therefore, we suffice to show $\partial\Pi_{AN}(s^*_{AC})/\partial s > 0$, which is equivalent to $s^*_{AC} < (\alpha - \sigma c)/(\alpha + \sigma c)$. By looking at the derivative of platform's profit under collusion with respect to s at $s = (\alpha - \sigma c)/(\alpha + \sigma c)$, we have

$$\frac{\partial\Pi_{AC}\left(s = \frac{\alpha-\sigma c}{\alpha+\sigma c}\right)}{\partial s} = -\frac{n}{4\sigma} \cdot \frac{\alpha(\alpha-\sigma c)^2}{4\sigma c} < 0 \iff s^*_{AC} < \frac{\alpha - \sigma c}{\alpha + \sigma c}, \tag{A.5}$$

which was what we wanted.

Second, we show $p^*_{AN} < p^*_{AC}$. It holds that $p_{AN}(s) < p_{AC}(s)$ for all $s \in (0, 1 - \sigma c/\alpha)$. Because both $p_{AN}(s)$ and $p_{AC}(s)$ are increasing function in s, there exists $s' > s^*_{AC}$ such that $p_{AN}(s') = p_{AC}(s^*_{AC})$ holds. Substituting $s = s'$ into $\partial\Pi_{AN}(s)/\partial s$ and then simplifying it with equation (12), we have

$$\frac{\partial\Pi_{AN}(s')}{\partial s} = -\frac{\beta^2 - \sigma^2}{4(1-s^*_{AC})^3\beta^2\sigma} \left[\begin{array}{l} (1-s^*_{AC})^2\alpha^2(\beta-\sigma) - (\beta+\sigma)\sigma^2 c^2 \\ +2(1-s^*_{AC})\alpha(s^*_{AC}\beta + \sigma)\sigma c \end{array} \right] < 0, \tag{A.6}$$

which implies that $s^*_{AN} < s'$. Because $p_{AN}(s)$ is increasing function, it follows that $p_{AC}(s^*_{AC}) = p_{AN}(s') > p_{AN}(s^*_{AN})$. The higher retail price under the collusive equilibrium directly leads the lower demand, consumer surplus, and social welfare (i.e., $q^*_{AN} > q^*_{AC}$, $CS^*_{AN} > CS^*_{AC}$, and $SW^*_{AN} > SW^*_{AC}$).

Third, we show $\pi^*_{AN} < \pi^*_{AC}$. It holds that $\pi_{AN}(s) < \pi_{AC}(s)$ for all $s \in (0, 1 - \sigma c/\alpha)$. The manufacturers' profit is decreasing in s. Because $s^*_{AN} > s^*_{AC}$, it follows that $\pi_{AN}(s^*_{AN}) < \pi_{AC}(s^*_{AC})$.

Lastly, we show $\Pi^*_{AC} \leq \Pi^*_{AN}$. As above, we consider $s = s' > s^*_{AC}$ such that $p_{AN}(s') = p_{AC}(s^*_{AC})$. The same retail price leads the same total output, which also implies the same gross revenue in the channel. Because s' is larger than s^*_{AC}, the platform gains the larger share of the gross revenue, that is, $\Pi_{AC}(s^*_{AC}) < \Pi_{AN}(s')$. Finally, from the definition of s^*_{AN}, the platform can obtain the greater profit by charging s^*_{AN} than s', that is, $\Pi_{AN}(s') \leq \Pi_{AN}(s^*_{AN})$. Therefore, it holds that $\Pi^*_{AN} > \Pi^*_{AC}$. □

Proof of Lemma 3.

First, we can show that $p^*_{WC} > p^*_{AC}$ and $q^*_{WC} < q^*_{AC}$ by simple calculations, which in turn imply that $CS^*_{WC} < CS^*_{AC}$ and $SW^*_{WC} < SW^*_{AC}$.

Next, we show that $\pi^*_{WC} > \pi^*_{AC}$ holds. There exists a unique $s'' \in (0, 1 - \sigma c/\alpha)$ such that $\pi^*_{WC} = \pi_{AC}(s'')$. Substituting $s = s''$ into $\partial \Pi_{AC}(s)/\partial s$, we have $\partial \Pi_{AC}(s'')/\partial s > 0$. Because $\Pi_{AC}(s)$ is concave, it holds that $s'' < s^*_{AC}$. Moreover, because $\pi_{AC}(s)$ is decreasing in s, it holds that $\pi_{AC}(s'') > \pi_{AC}(s^*_{AC})$. In sum, we can derive that $\pi^*_{WC} = \pi_{AC}(s'') > \pi_{AC}(s^*_{AC}) = \pi^*_{AC}$ holds.

Lastly, we show that $\Pi^*_{WC} < \Pi^*_{AC}$ holds. Set s' such that $s' = (p^*_{WC} - w^*_{WC})/p^*_{WC}$. If $p_{AC}(s') < p^*_{WC}$ holds, then we have

$$p^*_{WC} q(p^*_{WC}) < p_{AC}(s') q(p_{AC}(s')) \tag{A.7}$$

by the concavity of $pq(p)$ in p and the inequality $\arg\max_p pq(p) < p_{AC}(s') < p^*_{WC}$, where $q(p) \equiv q_i(p, \ldots, p)$ for $i = 1, \ldots n$. This in turn implies that

$$\Pi^*_{AC} = \max_s \Pi_{AC}(s) \geq \Pi_{AC}(s) = \frac{p^*_{WC} - w^*_{WC}}{p^*_{WC}} p_{AC}(s') q(p_{AC}(s')) > \Pi^*_{WC}. \tag{A.8}$$

What remains to be shown is that $p_{AC}(s') < p^*_{WC}$. By the fact that $p^*_{WC} = (3\alpha + \sigma c)/4\sigma$, $w^*_{WC} = \frac{\alpha}{2\sigma} + \frac{c}{2}$, and $p_{AC}(s) = \{\alpha + \sigma c/(1 - s)\}/2\sigma$, we have

$$p^*_{WC} - p_{AC}(s') = \frac{\alpha(\alpha - \sigma c)}{4\sigma(\alpha + \sigma c)} > 0. \tag{A.9}$$

Thus, we have $p_{AC}(s') < p^*_{WC}$, which completes the proof. □

Proof of Proposition 2.

Here, we show that the monopoly platform always has an incentive to deter the formation of upstream collusion for all $s \in [0, 1]$.

First, let us consider the case in which the monopoly platform sets $s \neq s_N$ and upstream manufacturers sustain their collusion. The monopoly platform gains $\Pi_{AC}(s)$ for every period. If the platform deviates from charging s, the best deviation is to charge $s = s_N$ because manufacturers who observe any revenue-sharing rate different from s will start to play the stage-game Nash equilibrium strategy (i.e., $p_i = p_{AN}(s)$ for all i). This optimal deviation enables the platform to obtain $\Pi_{AN}(s_N) = \Pi^*_{AN}$ over subsequent future periods. From the result of Lemma 2, the following string of inequalities holds: $\Pi_{AC}(s) < \Pi^*_{AC} < \Pi^*_{AN}$. Therefore, the deviation to $s = s_N$ always improve the profit of the monopoly platform. In other words, the platform has an incentive to deter the upstream collusion by setting $s = s_N$.

Next, we look at the remaining case of $s = s_N$. As above, by deviating to $s = s_N \pm \varepsilon$ where ε represents an arbitrary small number, the monopoly platform can deter the cartel formation and then derive the profit of $\Pi_{AN}(s_N \pm \varepsilon) \simeq \Pi^*_{AN}$, which is strictly greater than $\Pi_{AC}(s_N)$. Thus, the platform has an incentive to deter the upstream collusion even for $s = s_N$.

In total, the monopoly platform always has an incentive to deter the upstream collusion for all $s \in [0, 1]$ □

Proof of Lemma 7

The derivative of $\delta_A(s)$ can be computed as

$$\delta'_A(s) = \frac{16\lambda\mu^2(1-\mu)^2(2-\mu)^2(2\lambda-\mu-\lambda\mu)(2-\mu-\lambda\mu)}{\{(1-s)(2-\mu)^4(2-\mu-\lambda\mu) - 16\lambda(1-\mu)^2(2\lambda-\mu-\lambda\mu)\}^2}, \tag{A.10}$$

which is greater than zero when all the stage-game Nash equilibrium outcomes in Table 4 are positive (i.e., $2\lambda - \mu - \lambda\mu > 0$ and $2 - \mu - \lambda\mu > 0$). □

Proof of Lemma 8

$$\begin{aligned} \Pi^C(s,s) - \Pi^N_d(s,s) &= \frac{s}{2(1+\lambda)(1+\mu)} - \frac{2s(1-\mu)}{(1+\lambda)(2-\mu)^2(1+\mu)} \\ &= \frac{s\mu^2}{(1+\lambda)(2-\mu)^2(1+\mu)} > 0 \end{aligned} \tag{A.11}$$

□

Proof of Proposition 4

Here, we will show $\delta^{**}_A < \delta^{**}_W$. To this end, because Lemma 7 (i) shows that $\delta_A(s)$ is an increasing function and Lemma 7 (ii) shows $\delta_A(s^{**}_N) = \delta^{**}_W$, it suffices to show that neither platform has an incentive to deviate from $(s_1, s_2) = (s^{**}_N - \varepsilon, s^{**}_N - \varepsilon)$.

Let us first consider $(s_1, s_2) = (s^{**}_N, s^{**}_N)$, in which competing platforms gain $\Pi^C(s^{**}_N, s^{**}_N)$ in every period. If a platform deviates from charging s^{**}_N in a period, then manufacturers play the

Nash equilibrium strategy in that period. Thus, the best deviation is charging $s_N^{**} \pm \varepsilon$. The deviating platform will gain $\Pi_d^N(s_N^{**} \pm \varepsilon, s_N^{**}) = \Pi_{AN}^{**} - \varepsilon'$ in that period, which is strictly less than the payoff that the platform would gain if it had not deviated, as shown in Lemma 8 (i.e., $\Pi_d^N(s_N^{**}, s_N^{**}) < \Pi^C(s_N^{**}, s_N^{**})$). Because even the best deviation does not pay, neither platform has an incentive to deviate from $(s_1, s_2) = (s_N^{**}, s_N^{**})$ in every period.

Due to continuity of all variables, this argument holds for a neighborhood of $(s_1, s_2) = (s_N^{**}, s_N^{**})$. Consequently, it turns out that neither platform has an incentive to deviate from $(s_1, s_2) = (s_N^{**} - \varepsilon, s_N^{**} - \varepsilon)$ in every period. This completes the proof. □

Appendix B: Relaxation on deviations by monopoly platform

First, we derive the deviation payoff given a revenue-sharing rule s. Without loss of generality, we consider the deviation by manufacturer 1 in the second round, i.e., the other manufacturers charge the collusion price for a given s (i.e., $p_j = p_{AC}(s)$ for $j = 2, \cdots, n$). Then, the profit of manufacturer 1 can be expressed as presented below.

$$\pi_1^D(p_1) = \{(1-s)p_1 - c\}\{\alpha - \beta p_1 + (n-1)\gamma p_{AC}(s)\} \tag{B.1}$$

Solving $\partial \pi_1^D(p_1)/\partial p_1 = 0$ yields $p_{AD}(s) = \left\{\alpha + (n-1)\gamma p_C + \beta \frac{c}{1-s}\right\}/(2\beta)$. The deviation profit of manufacturer 1 is computed as $\pi_{AD}(s) = (1-s)(\beta+\sigma)^2 \left(\alpha - \sigma \frac{c}{1-s}\right)^2/(16\beta\sigma^2)$.

Next, let us characterize $\mathcal{S}(s^*)$. The condition for which neither manufacturer has deviation incentive given the platform will continue to choose s^* from the next period onwards is given by

$$\pi_{AD}(s) - \pi_{AC}(s) \leq \frac{\delta}{1-\delta}\{\pi_{AC}(s^*) - \pi_{AN}(s_{AN}^*)\}. \tag{B.2}$$

We use $\tilde{s}(\delta, s^*)$ to denote the value of revenue-sharing rule s such that condition (B.2) holds with equality. Given a pair of parameters $(\alpha, \beta, \sigma, c)$, one can interpret $\tilde{s}$ as a function of discount factor δ and the platform-preferred revenue-sharing rule s^*.

In the same vein, let $\delta_A(s, s^*)$ be the value of δ such that condition (B.2) holds with equality for any given s and the platform-preferred revenue-sharing rule s^*. That is, $\delta_A(s, s^*)$ is an inverse function of $\tilde{s}$ with respect to δ at a given s^*. Then, one can infer that condition (B.2) holds if and only if $\delta \geq \delta_A(s, s^*)$. We have the following lemma on the property of $\delta_A(\cdot, \cdot)$.

Lemma 9. *The following statements hold;*

(i) $\delta_A(s, s^*)$ *is increasing in* s^* *and decreasing in* s.

(ii) $\tilde{\delta}_A(s) := \delta_A(s, s)$ *is increasing in* s.

(iii) $\tilde{\delta}_A(s^*_{AN}) = \delta^*_W$.

Proof. (i) First, $\delta_A(s, s^*)$ is increasing in s^* simply because $\pi_{AC}(s^*)$ is decreasing. Second, $\delta_A(s, s^*)$ is decreasing in s because

$$\frac{\partial \pi_{AD}(s)}{\partial s} - \frac{\partial \pi_{AC}(s)}{\partial s} = \frac{\partial}{\partial s}\left\{(1-s)\left(\alpha - \sigma \frac{c}{1-s}\right)^2\right\}\left\{\frac{(\beta+\sigma)^2}{16\beta\sigma^2} - \frac{1}{4\sigma}\right\} < 0. \tag{B.3}$$

(ii) Formally, $\tilde{\delta}_A(s)$ is computed as follows.

$$\tilde{\delta}_A(s) = \delta_A(s, s) = \frac{\pi_{AD}(s) - \pi_{AC}(s)}{\pi_{AD}(s) - \pi^*_{AN}} \tag{B.4}$$

The derivative of $\tilde{\delta}_A(s)$ with respect to s is given by

$$\frac{\partial \delta_A(s)}{\partial s} = \frac{\left(\frac{\partial \pi_{AD}(s)}{\partial s} - \frac{\partial \pi_{AC}(s)}{\partial s}\right)\{\pi_{AD}(s) - \pi^*_{AN}\} - \{\pi_{AD}(s) - \pi_{AC}(s)\}\frac{\partial \pi_{AD}(s)}{\partial s}}{\{\pi_{AD}(s) - \pi^*_{AN}\}^2} \tag{B.5}$$

$$= \frac{\frac{1}{4\sigma}\left(\alpha + \sigma\frac{c}{1-s}\right)\left(\alpha - \sigma\frac{c}{1-s}\right)\frac{(\beta-\sigma)^2}{4\beta\sigma}\pi^*_{AN}}{\{\pi_{AD}(s) - \pi^*_{AN}\}^2}, \tag{B.6}$$

which is greater than 0. Therefore, $\tilde{\delta}_A(s)$ is an increasing function.
(iii) Substituting $s = s^*_{AN}$ into $\tilde{\delta}_A(s)$ directly yields $\tilde{\delta}_A(s^*_{AN}) = \delta^*_W$. □

Part (i) of Lemma 9 first shows that the collusion among manufacturers is harder to sustain as the platform charges a higher equilibrium revenue-sharing rule. Additionally, it implies that upstream collusion can be sustained more easily if the platform charges a revenue-sharing rule higher than the equilibrium level in a period. The intuition is that an increase in the future revenue-sharing rule s^* decreases the future value of sustaining collusion, whereas an increase in the current revenue-sharing rule decreases the current net value of deviating from the collusion.

This result also implies that collusion among manufacturers is sustainable if and only if $s \geq \tilde{s}(\delta, s^*)$. In other words, $\tilde{s}(\delta, s^*)$ is the smallest revenue-sharing rule at which the upstream collusion can be sustained.

Similarly, as shown in Lemma 9 (ii), there exists threshold $\tilde{s}_A(\delta)$ below which the collusion among manufacturers can be sustainable. That is, if $s^* \leq \tilde{s}_A(\delta)$, then the manufacturers can sustain their joint profit maximizing collusion. This result generates tension between fostering and hindering collusion in the platform's decision on s^*. Setting a high revenue-sharing rule for subsequent future periods hinders the formation of upstream collusion overall, but in each period, the platform should set a low revenue-sharing rule to hinder collusion.

In sum, given the equilibrium revenue-sharing rule s^*, a maximal set $\mathcal{S}^*(s^*)$ is determined as

$$\mathcal{S}^*(s^*) = \{s \in [0,1] \mid s > \tilde{s}(\delta, s^*)\}.. \tag{B.7}$$

Furthermore, given the maximal $\mathcal{S}^*$, the platform determines the equilibrium revenue-sharing rule s^* to satisfy

$$s^* = \arg\max_s \Pi_A(s, \mathcal{S}^*), \tag{B.8}$$

where

$$\Pi_A(s, \mathcal{S}^*) = \begin{cases} \Pi_{AC}(s) & \text{if } s \in \mathcal{S}^* \\ \Pi_{AN}(s) & \text{if } s \notin \mathcal{S}^* \end{cases} \tag{B.9}$$

Finally, a pair of the revenue-sharing rule and the maximal set $(s^*, \mathcal{S}^*(s^*))$ characterizes the equilibrium under consideration. Solving the above system, we can derive the platform-preferred revenue-sharing rate s^* and the critical discount factor δ_A^*, as shown in Proposition 5. □

References

Bernheim, B. D. and Whinston, M. D. (1985), 'Common marketing agency as a device for facilitating collusion', *The RAND Journal of Economics* **16**(2), 269–281.

Boik, A. and Corts, K. S. (2016), 'The effects of platform most-favored-nation clauses on competition and entry', *The Journal of Law and Economics* **59**(1), 105–134.

Choi, S. C. (1991), 'Price competition in a channel structure with a common retailer', *Marketing Science* **10**(4), 271–296.

Cooper, T. E. (1986), 'Most-favored-customer pricing and tacit collusion', *The RAND Journal of Economics* **17**(3), 377–388.

Dobson, P. W. and Waterson, M. (1996), 'Product range and interfirm competition', *Journal of Economics & Management Strategy* **5**(3), 317–341.

Foros, Ø., Kind, H. J. and Shaffer, G. (2017), 'Apple's agency model and the role of most-favored-nation clauses', *The RAND Journal of Economics* **48**(3), 673–703.

Gaudin, G. and White, A. (2014), 'On the antitrust economics of the electronic books industry', Available at SSRN: https://ssrn.com/abstract=2352495.

Huang, Z. (2017), 'The power of upstream contracting over downstream collusion', *Job Market Paper*.

Johansen, B. O. and Vergé, T. (2017), 'Platform price parity clauses with direct sales', *Working Paper Series* **No. 1**(17).

Johnson, J. P. (2017), 'The agency model and MFN clauses', *The Review of Economic Studies* **84**(3), 1151–1185.

Jullien, B. and Rey, P. (2007), 'Resale price maintenance and collusion', *The RAND Journal of Economics* **38**(4), 983–1001.

Lu, L. (2017), 'A comparison of the wholesale model and the agency model in differentiated markets', *Review of Industrial Organization* **51**(2), 151–172.

Maruyama, M. and Zennyo, Y. (2018), 'Platform most-favored-customer clauses and investment incentives', Available at SSRN: https://ssrn.com/abstract=3099305.

Neilson, W. S. and Winter, H. (1993), 'Bilateral most-favored-customer pricing and collusion', *The RAND Journal of Economics* **24**(1), 147–155.

Nocke, V. and White, L. (2007), 'Do vertical mergers facilitate upstream collusion?', *American Economic Review* **97**(4), 1321–1339.

Reisinger, M. and Thomes, T. P. (2017), 'Manufacturer collusion: Strategic implications of the channel structure', *Journal of Economics & Management Strategy* **26**(4), 923–954.

Rusu, C. S. (2016), 'Eturas: Of concerted practices, tacit approval, and the presumption of innocence', *Journal of European Competition Law & Practice* **7**(6), 396–398.