A Dynamic Analysis of Misconduct in Contemporary Social Context

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Abstract

Whereas many law literatures of punishment have addressed dynamic situation affected by the very punishment in the future, the extant economic analysis of misconduct heavily relies on static model. As the initial step toward the dynamic economic analysis of misconduct, our work models the life-time wealth of a worker who faces the opportunity of wrongdoing and the risk of being detected and suspended in order to characterize the deterrent effect of suspension on corporate misconduct. Under the centralized economy, our model predicts raising a worker's income stream unambiguously improves social welfare by relaxing the limited-liability constraint of a regulator. On the other hand, under the decentralized laissez-faire economy, our model suggests excessive compensation can reduce social welfare by squeezing production. To achieve the efficient level of production, corporate sanction, contingent on the level of a worker's compensation, is mandatory. As opposed to the existing literature, our model even rationalizes the government's subsidy and bailout in order to fix the shrink of production owing to excessive compensation.

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1. Introduction

Law scholars and financiers have tackled corporate crime matters in many ways (See e.g. Edgerton (1927), Muller (1957), Elkins (1976), Orland (1980), Coffee (1981), Fisse (1983), Bucy (1991), Fisse & Braithwaite (1993)). One of the most promising ways is economic analysis that seems to influence corporate crime policy significantly in the U.S. (See e.g. Arlen (1994), Arlen & Kraakman (1997), Arlen (2012)) and other countries (See Arlen (2020) and Arlen & Buell (forthcoming)). Under the unignorable influence of the canonical work of Becker (1968), most of law and economics literatures have chosen monetary sanction as the first choice of formal sanction to be imposed on rogue employee for deterrence. This choice seems very persuasive and even irrefutable, if we analyze static and fragmented world.

However, many law literatures of punishment have focused not only on static situation but on dynamic situation caused by the very punishment would be imposed on wrongdoers. They demonstrate the prospective impact of punishment in multiple ways. For example, Beccaria (1764), the god father of modern criminal law, demonstrated the future impact of public exposure of forced labor in his famous "On Crime and Punishment." Rousseau (1755) who illustrated the function of state power as a clue to solve coordination problem in dynamic game situation, might overlap his interest with a recent research of the "expressive function" of criminal punishment which focuses on the power of punishment that changes the behavior of potential perpetrators (McAdams, 2015). We may add Hegelian theory of punishment to the list which puts importance on cognitive assurance function of criminal punishment through rebuilding the confidence of legal stability of citizens who are shocked by crime and start to suspect the effectiveness of law (See Hegel (1820) and Jakobs (1997)).

In addition to their dynamic view, it is worth demonstrating that in modern society people live in the integrated world where their lives in corporation intrinsically connect to their lives in other social domains. It means that when we treat misconduct in corporation, it is better to take account in the influence of sanctions upon individuals and corporations relating to the misconduct in broader social context, because in our society most people earn their living expenses from corporations or other forms of organization. Therefor it seems us that there is a gap between previous economic analysis of punishment and legal literature of punishment. We wonder if many economic analyses may fail to take account in important feature of criminal punishment, that is criminal punishment works in dynamic and integrated world that will be or was influenced by the very punishment itself.[3] As we will show, this gap leads to underestimate the efficiency of imprisonment and stigmatizing effect of punishment and ultimately to understand the severity of current imprisonment punishment, which works in free and open society where many people earn their life through time-based income, difficultly.[4] In this article we take account in this inter-constructive relationship between punishment and the dynamic situation surrounding the punishment through dynamic economic analysis of corporate punishment. This work will bridge the current gap described above and deepen the understanding of punishment, which ultimately leads better understanding of imprisonment in general.

Among various types of misconduct, we focus on white-collar crimes by executive workers, as the initial step toward the dynamic economic analysis of punishment. Because executive workers earn relatively high compensation, the suspension of their future business through

[3] Francesco Guala, who defines institutions as "rules-in-equilibrium" in dynamic games, demonstrates a constructive character of legal rules (Guala, 2016).
[4] As Foucault illustrated, modern punishment measures like imprisonment in correction facility, had been formed in emerging modern society where "good" labor force became keenly demanded through industrialization (Foucault, 1977). Although prison comes to serve for different function of incapacitation after the "late modernity" (Garland, 2001) .

imprisonment or license removal is particularly relevant for deterring their crimes. Motivated by this conjecture, we incorporate the suspension of future economic activity into the model of corporate misconduct by Arlen & Kraakman (1997) as the primary method of criminal punishment on a wrongdoer. In our model, a worker has the opportunity of committing a wrong every period. A worker uses a threshold (reservation value) strategy to determine whether to commit a wrong or work prudently. A regulator monitors a worker stochastically, and deprives him of future economic activity when his wrongdoing is detected. Depriving a wrongdoer of his opportunity to work in the future relaxes the limited-liability constraint for a regulator and enhances the capacity of punishment, as it allows a regulator to penalize a wrongdoer effectively even if a wrongdoer has had little wealth or consumed all the existing wealth before being detected. Then, the reservation value for committing a wrong is, ceteris paribus, greater in the presence of suspension than the one characterized by Arlen & Kraakman (1997). As the wealth to be lost by a wrongdoer through punishment depends on his income stream in the future, the model becomes dynamic by nature. Although our dynamic model becomes intrinsically complex, it gives us useful implications under certain special, but reasonably realistic cases.

We derive four major outcomes from our model. First, we find intensifying suspension and raising a worker's income stream unambiguously improve social welfare under the centralized economy or economy with corporate sanction. Both strengthen the effect of monitoring by imposing severer sanction upon the detection of wrongdoing. Second, under cheap monitoring cost, there is substitutional relationship between the efficient level of monitoring and the size of sanction. In particular, the efficient level of monitoring decreases in the level of a worker's income stream and the intensity of suspension. On the same ground, the efficient level of monitoring decreases in a firm's age, if a worker's compensation grows or suspension intensifies over time. Third, under the decentralized laissez-faire economy, excessive compensation can reduce social welfare by squeezing production. Under the unregulated economy, a firm's choice of production decreases in a worker's compensation, which is a part of production cost incurred by a firm. This result is contrastive to the favourable impact of a worker's compensation under the regulated economy. Last, the efficient level of corporate sanction can be both positive and negative, depending on the level of a worker's compensation. If a worker's compensation is sufficiently large, the efficient level of corporate sanction becomes negative. What is more, the size of optimal subsidy can increase in the level of a worker's compensation, because the optimal frequency of detecting a wrong can decrease in a worker's compensation, which needs to be compensated to sustain the expected level of subsidy.

Our results imply a key deterrent to corporate misconduct is a worker's income stream, affected by his prospective compensation. On the one hand, raising income stream improves social welfare unambiguously under the regulated economy. On the other hand, excessive compensation can aggravate efficiency under the unregulated economy. To achieve an efficient outcome under the decentralized economy, corporate sanction is essential for inducing a firm to internalize the social harm of business activity attributed to its worker's wrongdoing. The size of sanction depends on the level of a worker's compensation. In fact, it can be even negative if a worker's compensation is sufficiently large. Thus, if a regulator aims to achieve social optimum by raising a worker's compensation as much as possible, there is possibility that a regulator subsidizes a firm when a wrongdoing is detected, because a firm pays too much cost for its production without subsidy. This result is contrastive to the proposal of penalizing a firm upon the detection of wrongdoing by Arlen & Kraakman (1997).

Moreover, our analysis suggests the efficient level of monitoring, or probability of detecting a wrong, decreases in a worker's income stream and the intensity of suspension, at least when monitoring cost is relatively cheap. Although monitoring cost can be high in the presence of severe information asymmetry between a firm and a regulator, it can be small under the modern system of internal governance and the scheme designed to encourage self-reporting a wrong. For example, public companies have enhanced financial disclosure since the Sarbanes–Oxley (SOX) Act of 2002, while the Department of Justice (DOJ) has encouraged whistle-blowing and self-reporting through Deferred Prosecution Agreements (DPA) and leniency program. This result suggests excessive monitoring is unnecessary in the presence of high compensation under the modern system of internal governance and the incentive scheme that encourages self-reporting. On the same ground, this result motivates the life-cycle theory of efficient monitoring: a younger firm should be monitored more frequently than an elder firm if a firm's compensation grows over time.

Our work first contributes to the economic analysis of law enforcement by explicitly incorporating the suspension of future economic activity into the theory of deterrence. The existing economic theory of deterrence builds on idea that monitoring, or the probability of detecting a wrong, and the severity of sanction jointly deter a crime (Garoupa, 2002). Becker (1968) argues maximizing fine is the optimal way of deterring a crime, considering it minimizes the need for costly monitoring whereas it is a costless transfer for a society. However, maximizing fine is not so effective when a wrongdoer has limited wealth, suggesting the difficulty of efficient deterrence through monetary sanction. In response to this problem, some of the following studies emphasize the importance of observing an idiosyncratic characteristic of an individual and a wrong. They show this limited-liability constraint is unlikely to bind at least for certain individuals and crimes. For example, Shavell (1987) argues expensive fines are often unnecessary for a petty crime committed by a poor individual, because a private benefit from wrongdoing is likely to be large relative to a harm imposed on a society[5]. In another example, Bebchuk & Kaplow (1993) find the optimal fines should be less than maximal for individuals who are less difficult to apprehend. Moreover, others argue substituting fines with monitoring is not as costly as thought by Becker (1968). For example, Kaplow & Shavell (1994) find a regulator can raise monitoring intensity without spending resources by pre-committing to offer less severe penalties to those who self-report harm-producing actions. They suggest raising the probability of deterring a wrong through a leniency program can reduce fines without sacrificing deterrent effect. Unlike these studies, our work rather questions the presence of limited-liability constraint itself by reconsidering the definition of an individual's "wealth." As admitted by Becker himself, the amount of time available to an individual multiplied by his wage is a significant component of his "life-time wealth" (Becker, 1965). In this spirit, our work models the life-time wealth of an individual who faces the opportunity of wrongdoing and the risk of being detected and suspended in order to characterize the deterrent effect of removing one's future life-time through suspension. This feature is distinct from the earlier literature of non-monetary sanctions (See e.g. Block & Lind (1975), Polinsky & Shavell (1999)).

Moreover, our policy implication contributes to the ongoing discussion of executive compensation regulation. Specifically, our results suggest lowering the level of executive compensation weakens the deterrent effect of suspension, which is contrastive to the recent regulatory trend of restricting executive compensation level in response to the Global Financial Crisis (GFC). Indeed, the Dodd-Frank Act (DFA) of 2010 requires the Securities and Exchange

[5] Researchers in corporate law and finance rather attempt to rationalize the limited liability of a corporate controller (e.g., an owner-manager) by arguing it raises capital supply in financial market, precludes the adverse selection of poor investors, and reveals valuable information to public. See Easterbrook and Fischel (1985), Winton (1993), and Asai (2020), for example.

Commission (SEC) to implement rules mandating U.S. companies to adopt clawback policies, whereas the Capital Requirements Directive (CRD IV) caps the bonuses of executive bankers in E.U. Our work complements the existing research by assessing the effect of compensation *level* on business conduct, rather than the effect of pay *structure*, which has been the main focus of the recent work on executive compensation (See e.g. Fahlenbrach & Stulz (2011), DeYoung et al. (2013), Brandão Marques et al. (2014), Asai (2016), Albuquerque et al. (2019)).

2. Model

We make a model of corporate wrongdoing in the spirit of Arlen & Kraakman (1997). As in Arlen & Kraakman (1997), a risk neutral firm hires risk neutral agents to produce a product. Each employee is penniless, producing one unit of the product per period. Total firm production in period t, which is equivalent to total employment per period, is given by q_t ($q_t > 0$).[6] The revenue of the firm's production is given by $V(q_t)$. We assume V satisfies Inada conditions. Although it is easiest to interpret q_t as the number of employees, q_t can be considered as the ``size" of the firm (e.g., the level of investment or external capital) under the condition that the number of employees is proportional to the firm's size.

Each firm incurs the cost from production every period which has a monetary value of k ($k \geq 0$). Departing from Arlen & Kraakman (1997), we assume each firm pays base salary, w_t, to each employee ($w_t \geq 0$). The salary is exogenous and immediately consumable for the fraction of $1 -$

[6] Polinsky & Shavell (1993) also analyze corporate misconduct while endogenizing a firm's output level.

α $(0 \leq \alpha < 1)$; $1 - \alpha$ of w_t cannot be transferred from an employee to a third party after the employee receives it.

Each employee has the opportunity to commit a wrong every period. The cost to society of each wrong is H. The benefit to an employee of wrongdoing is given by b, which varies across employees. It is independent and identically distributed over time. Neither the regulator nor the firm knows any individual's b. Instead, the regulator and firm know the probability density function and the cumulative distribution function of b, which is given by $r(b)$ and $R(b)$, respectively. The firm also receives a surplus, B, from wrongdoing.

Wrongdoing is socially optimal if the net benefit of wrongdoing exceeds the social cost of the wrong:

$$b + B > H.$$

On the other hand, an employee commits a wrong when his personal benefit exceeds the cost of wrongdoing.

In the absence of non-negative wealth constraint, the planner can induce employees to refrain from suboptimal wrongdoing by imposing the sanction equal to, $H - B$, divided by the probability of detection. This sanction, however, is unlikely to be paid by each employee in the presence of limited liability, because he does not have sufficient wealth.

To deter wrongdoing, the planner has to rely on non-monetary sanctions, such as imprisonment and suspension. For the ease of exposition, we discuss the model in the context of imprisonment, but such imprisonment can be generalizable to any type of suspension. As suggested by the previous research, such non-monetary sanction can increase the amount of penalties by removing the opportunity of working in the future.

To characterize this effect formally, we model the life-time wealth of a worker who faces the opportunity of wrongdoing and the risk of being detected and suspended. Each employee is hired by the firm each period until terminal period T unless he is in "prison."[7] In period t, each employee knows the probability of being detected at the end of the period, $p(M_{t'})$, where $M_{t'}$ is the planner's monitoring for $\forall t' \geq t$ ($M_{t'} > 0, \forall t' \geq t$). We consider that αw_t is confiscated when his wrongdoing is detected, because it is transferrable. If he does not commit a wrong, his value is the discounted value of being out of prison in $t+1$ plus the current salary w_t. Then he commits a wrong if and only if:

$$b - p(M_t)\alpha w_t + \beta\Big(\big(1 - p(M_t)\big)W_{t+1} + p(M_t)U_{t+1}\Big) \geq \beta W_{t+1}, \text{(1)}$$

where β is the discount factor ($0 \leq \beta < 1$). Therefore, his value of being out of prison in t, W_t, is

$$W_t = \int_{p(M_t)(\alpha w_t + \beta(W_{t+1} - U_{t+1}))}^{\infty} \big(b - p(M_t)(\alpha w_t + \beta(W_{t+1} - U_{t+1})\,)\big)r(b)db$$

$$+ w_t + \beta W_{t+1}. \text{(2)}$$

On the other hand, his value of being in prison at the beginning of t is the weighted average of the value when he is released from prison and the discounted value of U_{t+1}.

$$U_t = \lambda_t W_t + (1 - \lambda_t)\beta U_{t+1}, \text{(3)}$$

where λ_t is the probability of being released from prison. Note that the probability of being released from prison, λ_t, captures the intensity of imprisonment. If it is zero, that is, if an employee cannot escape from prison, his value of being in prison is zero. In this case, his cost of committing a wrong is maximized. If it is one, that is, if he can always escape from prison, his value of being

[7] Here, we refer to any type of confinement, not necessarily imprisonment.

in prison is equal to the value of being out of prison. Then his cost of committing a wrong is minimized and equal to the one in the absence of imprisonment.[8] Given the value of an employee after the terminal period $\{W_{T+1}, U_{T+1}\}$, we can recursively solve $\{W_{t'}, U_{t'}\}$ for $\forall t' \geq t$ as a function of variables $\{M_{t''}, \lambda_{t''}, w_{t''}\}$ for $\forall t'' \geq t'$.

The social welfare induced by the firm is the sum of current and discounted future surplus. Then, it is characterized by:

$$S = \sum_{t=0}^{T} \beta^t \left\{ \begin{array}{c} V(q_t) \\ -q_t \left[\begin{array}{c} k + cM_t \\ + \int_{p(M_t)(\alpha w_t + \beta(W_{t+1} - U_{t+1}))}^{\infty} (H - b - B) r(b) db \end{array} \right] \end{array} \right\},$$

subject to $W_{T+1} = \overline{W}, U_{T+1} = \overline{U}$, (4)

where cM_t is the monitoring cost per employee incurred by the society ($c \geq 0$). For the rest of the paper, we denote equilibrium outcome by superscript $*$.

3. Social planner's problem

We start from considering the equilibrium under which the social planner determines the firm's employment and the level of monitoring. Then, the equilibrium is defined by:

$$S^* = \max_{\{q_t, M_t\}} S \text{ subject to } W_{T+1} = \overline{W}, U_{T+1} = \overline{U}.$$

[8] In this paper, we consider the cost of imprisonment is zero and the probability of being released from prison is exogenous.

To clarify the link to Arlen & Kraakman (1997), we start from solving the social planner's problem when there is no imprisonment, that is, $\lambda_t = 1, \forall 0 \le t \le T$. Then, $W_{t+1} = U_{t+1}, \forall 0 \le t \le T$. The first order conditions with respect to $\{q_t, M_t\}$ are:

$$V'(q_t) = k + cM_t + \left(H - E\left[b|b \ge \hat{b}_t\right] - B\right)\left(1 - R\left(\hat{b}_t\right)\right), \text{(5)}$$

$$\underbrace{p'(M_t)\alpha w_t r\left(\hat{b}_t\right)}_{\text{dec. in wrongdoers in t}} \ \hat{a}_t = c, \text{(6)}$$

where $\hat{a}_t = H - p(M_t)\alpha w_t - B$, $\hat{b}_t = p(M_t)\alpha w_t$. $\hat{a}_t$ represents the social loss of the marginal wrongdoer. $\hat{b}_t$ represents the reservation value for wrongdoing. The first condition suggests that employment level is optimal when the marginal social benefit of additional employment, $V'(q_t)$, equals the marginal social cost of employment. The second condition means that monitoring level is optimal when the marginal social cost of monitoring per worker, c, equals the marginal social benefit of monitoring per worker. These conditions suggest the dynamics of optimal $\{q_t, M_t\}$ are driven by the salary of an employee w_t. If $w_t = w$ where w is some constant, they are equivalent to those in the static version proposed by Arlen & Kraakman (1997).

Unlike the static version, the wealth of the wrongdoer that he loses when his wrongdoing is detected includes the discounted net value of being out of prison, $\beta(W_{t+1} - U_{t+1})$. Correspondingly, the social loss of the marginal wrongdoer is represented by

$$\tilde{a}_t = H - p(M_t)\left(\alpha w_t + \beta(W_{t+1} - U_{t+1})\right) - B.$$

Also, the reservation value for wrongdoing is represented by

$$\tilde{b}_t = p(M_t)\left(\alpha w_t + \beta(W_{t+1} - U_{t+1})\right).$$

As a result, the wrongdoer's reservation value is larger than the one in the static version.

Whereas the optimal monitoring level is hard to characterize in the dynamic model, the optimal employment is characterized in the similar manner as in the static version. Specifically, it satisfies:

$$V'(q_t) = k + cM_t^* + \left(H - E\left[b|b \geq \tilde{b}_t^*\right] - B\right)\left(1 - R\left(\tilde{b}_t^*\right)\right), \quad (7)$$

where $\tilde{b}_t^*$ is $\tilde{b}_t$ evaluated at the optimal monitoring level M_t^*. Then, we claim the following:

Proposition 1: Under the social planner's problem, q_t^* satisfies equation (7).

Proof: Obvious.

For the following part, we describe special cases that allow us to acquire analytical implications.

3-1. Social planner's problem under stationary model

We first consider a stationary version in which all the parameters and control variables are time-invariant while $T = \infty$. Under this condition, the value of an employee becomes also time-invariant. Specifically, we solve the following value function.

$$S^* = \max_{\{q,M\}} \sum_{t=0}^{\infty} \beta^t \begin{Bmatrix} V(q) \\ -q\begin{bmatrix} k + cM \\ + \int_{p(M)(\alpha w + \beta(W-U))}^{\infty} (H - b - B) r(b) db \end{bmatrix} \end{Bmatrix}.$$

The first order conditions with respect to $\{q\,, M\}$ are:

$$V'(q) = k + cM + \left(H - E\left[b|b \geq \tilde{b}\right] - B\right)\left(1 - R\left(\tilde{b}\right)\right), \quad (8)$$

$$\underbrace{\left(p'(M)(\alpha w + \beta\Gamma W) + p(M)\beta\Gamma(\partial W/\partial M)\right)r\left(\tilde{b}\right)}_{\text{dec. in wrongdoers}} \tilde{a} = c, \quad (9)$$

where $\tilde{a} = H - p(M)(\alpha w + \beta\Gamma W) - B$, $\tilde{b} = p(M)(\alpha w + \beta\Gamma W)$, and $\Gamma = (1-\lambda)(1-\beta)/(1-(1-\lambda)\beta)$. In this version, the wealth of the wrongdoer that he loses when his wrongdoing is detected includes the discounted net value of being out of prison, $\beta\Gamma W$.

As indicated above, the discounted net value of being out of prison is simplified to a linear function of the value of being out of prison W under the stationary model. The coefficient Γ captures the intensity of suspension. If the probability of being released from a prison is 1, $\Gamma = 0$ as there is no penalty of being detected. However, if the probability of being detected is 0, $\Gamma = 1$ as a wrongdoer completely loses the opportunity of future economic activity, suggesting he would lose the entire value of being out of prison W.

We now investigate the welfare implication of this special case. In particular, we assess the effects of parameters on social welfare. We claim:

Proposition 2: Under the stationary social planner's problem, $\{q^*, M^*\}$ satisfies equations (8) and (9) evaluated at $\{q, M\} = \{q^*, M^*\}$. Moreover, $dS^*/d\lambda \leq 0, dS^*/dw \geq 0$.

Proof: See Appendix.

Raising the salary of an employee and lowering the probability of releasing a prisoner is welfare-enhancing. By raising the wealth that an employee loses when his wrongdoing is detected, the planner can deter wrongdoing with less resources.

The policy implication from this result is perhaps surprising. Because w raises social welfare unambiguously, the planner should raise the salary of an employee as much as possible. Despite the criticism against the relatively high salary of a potential wrongdoer, our analysis rather rationalizes it as an efficient outcome.

3-2. Social planner's problem under zero monitoring cost

We reconsider the original dynamic model, but we now assume the cost of monitoring is zero ($c = 0$). We also assume the economy stays at the stationary equilibrium since $T + 1$. Then, such condition can be described by setting $W_{T+1} = W^*$ and $U_{T+1} = U^*$, where W^* and U^* are W and U evaluated at some equilibrium of the social planner's problem under the stationary version. We denote the stationary level of wage and releasing probability by w and λ, respectively. Trivially, if every w_t and λ_t is equal to w and λ, respectively, the model becomes equivalent to the stationary model with $c = 0$.

When monitoring is costless, the monitoring level is set so that the net social harm, $\tilde{a}_t$, becomes zero for every t. This means the planner raises monitoring level until the social loss of the marginal wrongdoer becomes zero. Indeed, if monitoring exceeded that level, the marginal wrongdoer would raise social welfare by wrongdoing and the planner could raise social welfare by reducing monitoring level to attract more wrongdoers. On the other hand, if monitoring fell below that level, the marginal wrongdoer would reduce social welfare by wrongdoing and the planner could raise social welfare by strengthening monitoring level to reduce wrongdoers without incurring cost. It therefore suggests:

$$\tilde{a}_t = H - p(M_t)\big(\alpha w_t + \beta(W_{t+1} - U_{t+1})\big) - B = 0. \quad (10)$$

Equation (10) suggests the reservation value for wrongdoing, $\tilde{b}_t$, is $H - B$. Then, the first order condition with respect to $\{q_t\}$ is:

$$V'(q_t) = k + (H - E[b|b \geq H - B] - B)\big(1 - R(H - B)\big). \quad (11)$$

This result suggests the optimal firm size is constant to t. Hence, $q_t^* = q^*$ for all t.

We now investigate the dynamics of monitoring level for this special case. In particular, we assess the effect of monotone trend of a parameter on the optimal level of monitoring. Because the monotone trend of a parameter induces the monotone trend of the wealth that an employee loses when his wrongdoing is detected, it induces the monotone trend of optimal monitoring level as the planner substitutes the wealth that an employee loses and the level of monitoring to achieve efficient deterrence against wrongdoing. Specifically, we claim the following.

Proposition 3: Under the social planner's problem under zero monitoring cost, $\{q_t^*, M_t^*\}$ satisfies equations (10) and (11) evaluated at $\{q_t, M_t\} = \{q_t^*, M_t^*\}$. Then, q_t^* is constant and invariant to salary and releasing probability at any t. Also, if w_t weakly increases (decreases) in t while λ_t weakly decreases (increases) in t, $q_t^* = q^*$ and M_t^* weakly decreases (increases) in t. Under the stationary model, $dq^*/d\lambda = 0, dq^*/dw = 0, dM^*/d\lambda \geq 0, dM^*/dw \leq 0$.

Proof: The constancy of q_t^* and its independence from salaries and releasing probability is obvious from equation (11). See Appendix for the rest.

Overall, this result indicates substitutional relationship between optimal monitoring and non-monetary sanction. If salary rises or releasing probability decreases, the optimal level of non-monetary sanction rises. Consequently, there is less monitoring required to achieve the efficient outcome.

This result also suggests as compensation grows (declines) while releasing probability declines (grows), the optimal level of monitoring declines (rises). It also suggests the optimal monitoring level depends on the life cycle of a firm. A regulator wants to raise monitoring intensity for a younger firm if a firm's compensation grows while releasing probability decreases by firm age, whereas the opposite should be valid if a firm's compensation falls while releasing probability increases by firm age.

4. Decentralized economy

In this section, we analyze the economy in which the firm's shareholder determines the firm's size, q_t. Let M_t^* be the optimal level of monitoring set by the social planner as in the planner's problem. Then, the shareholder maximizes her value by solving:

$$\max_{\{q_t\}} \sum_{t=0}^{T} \beta^t \left\{ \begin{array}{c} V(q_t) \\ -q_t \left[\begin{array}{c} k + w_t \\ + \int_{p(M_t^*)\left(\alpha w_t + \beta\left(W_{t+1}^* - U_{t+1}^*\right)\right)}^{\infty} (-B) r(b) db \end{array} \right] \end{array} \right\},$$

where $\{W_t^*, U_t^*\}$ is $\{W_t, U_t\}$ evaluated at $M_t = M_t^*$. The first order condition with respect to q_t is:

$$V'(q_t) = k + w_t + (-B)\left(1 - R\left(\tilde{b}_t^*\right)\right), \quad (12)$$

where $\tilde{b}_t^*$ is $\tilde{b}_t$ evaluated at $M_t = M_t^*$. Then, we claim the following:

Proposition 4: Under the decentralized economy, q_t^* satisfies equation (12).

Proof: Obvious.

Unlike the social planner's problem, the firm's marginal cost of hiring an employee increases in the salary of an employee in the decentralized economy. Moreover, it ignores the harm incurred by the society and the benefit received by an employee from wrongdoing. Further, it does not account for the monitoring cost incurred by the society.

4-1. Decentralized economy under special cases

When the shareholder's surplus from wrongdoing is zero ($B = 0$), the shareholder's marginal cost of hiring only depends on the salary of an employee w_t. As a result, the firm size chosen by the shareholder, q_t^*, does not interact with the monitoring level chosen by the planner, M_t^*.

In addition, when monitoring cost is zero ($c = 0$), the planner sets M_t^* such that $\tilde{a}_t^* = 0$ and $\tilde{b}_t^* = H - B$ as discussed in the previous section. Because the expected benefit of the shareholder from wrongdoing is now fixed to $B(1 - R(H - B))$, the shareholder's marginal cost of hiring only depends on the salary of an employee w_t and some fixed cost associated with firm size k. Again, the firm size chosen by the shareholder, q_t^*, does not interact with the monitoring level chosen by the planner, M_t^*.

These two observations allow us to claim the following.

Proposition 5: Suppose $B = 0$ or $c = 0$ under the decentralized economy. $dq_t^*/d\lambda_t = 0, dq_t^*/dw_t < 0, dq_{t'}^*/d\lambda_t = 0, dq_{t'}^*/dw_t = 0, \forall t' \neq t$. Moreover, $S^* \to 0, q_t^* \to 0$, as $w_t \to \infty, \forall t \geq 0$. Similarly, under the stationary model, $dq^*/d\lambda = 0, dq^*/dw < 0$, and $S^* \to 0, q^* \to 0$, as $w \to \infty$.

Proof: Obvious from equation (12) regarding the result about q_t^*. Also, due to Inada condition, $q_t^* \to 0$ as $w_t \to \infty$ and the right-hand-side diverges to ∞.

This result suggests, under certain conditions, as compensation grows, the firm size declines whereas the welfare of the economy starts to decline at some point. In the limit case under which the salary of an employee is infinitely large, the welfare of the economy converges to zero, which is the lower bound of the welfare of the economy under the planner's problem. Thus, excessive salary can reduce social welfare under this setup, which is contrastive to the implication from the social planner's problem.

5. Decentralized economy with corporate sanction

Keeping the setup of the decentralized economy, we now allow the planner to introduce corporate sanction, F_t, which is the transfer from the firm to the planner when corporate wrongdoing is detected. Let M_t^* be the optimal level of monitoring set by the social planner as in the planner's problem. Then, in the presence of this sanction, the shareholder's problem is:

$$\max_{\{q_t\}} \sum_{t=0}^{T} \beta^t \left\{ \begin{matrix} V(q_t) \\ -q_t \left[\begin{matrix} k + w_t \\ + \int_{p(M_t^*)(\alpha w_t + \beta(W_{t+1}^* - U_{t+1}^*))}^{\infty} (p(M_t^*)F_t - B) r(b) db \end{matrix} \right] \end{matrix} \right\}.$$

The first order condition with respect to q_t is:

$$V'(q_t) = k + w_t + \left(E[p(M_t^*)F_t | b \geq \tilde{b}_t^*] - B\right)\left(1 - R\left(\tilde{b}_t^*\right)\right), \quad (13)$$

where $\tilde{b}_t^*$ is $\tilde{b}_t$ evaluated at $M_t = M_t^*$. Considering the first order condition with respect to $\{q_t\}$ for the planner's problem (see equation (7)),[9] the optimal corporate sanction, F_t^*, is:

$$F_t^* = (H - b)/p(M_t^*) + (cM_t^* - w_t)/\left(p(M_t^*)\left(1 - R\left(\tilde{b}_t^*\right)\right)\right). \quad (14)$$

Alternatively, if the planner does not observe the benefit of the employee who commits wrongdoing, the planner can set:

[9] The planner achieves the social optimum by setting F_t such that

$$E\left[p(M_t^*)F_t \middle| b \geq \tilde{b}_t^*\right] - B = (cM_t^* - w_t)/\left(1 - R\left(\tilde{b}_t^*\right)\right) + H - E\left[b \middle| b \geq \tilde{b}_t^*\right] - B.$$

$$F_t^* = \left(H - E\left[b|b \geq \tilde{b}_t^*\right]\right)/p(M_t^*) + (cM_t^* - w_t)/\left(p(M_t^*)\left(1 - R\left(\tilde{b}_t^*\right)\right)\right). \quad (15)$$

In summary, we claim the following.

Proposition 6: Under the decentralized economy, q_t^* satisfies equation (13). At $F_t = F_t^*$ set in accordance with equation (14) or (15), the planner achieves the optimal level of firm size and monitoring under the social planner's problem.

Proof: Obvious from the above argument.

This result suggests corporate sanction is required even if a wrongdoer is optimally monitored. The major difference between this model and Arlen & Kraakman (1997) is the consideration of an employee's salary. If everything else equal, an employee's salary is negatively associated with the optimal level of corporate sanction. Then, the policy implication from this result is perhaps surprising. To see how, let us consider the stationary equilibrium. Because the salary of an employee raises social welfare unambiguously, a planner should raise the salary of an employee as much as possible. Moreover, because the optimal level of corporate sanction decreases in the salary of an employee ceteris paribus, the optimal level of corporate sanction could be even negative. Thus, Proposition 6 suggests a firm might be subsidized when wrongdoing is detected. Despite the criticism against the bail out of a failing firm owing to corporate misconduct and the high salary of its employees, our analysis provides the possibility of rationalizing both observations as an efficient outcome.

5-1. Decentralized economy with corporate sanction under special cases

We can formally verify this conjecture by focusing on the stationary case in which monitoring cost is zero. In this case, the reservation value for wrongdoing is fixed such that $\tilde{b}^* = H - B$, so

M^* decreases in w as the planner substitutes monitoring intensity with the size of wealth lost by a wrongdoer upon detection. When the salary is sufficiently large, the firm is subsidized when wrongdoing is detected. Then, the size of the subsidy increases as monitoring intensity decreases.

Proposition 7: Suppose the stationary model under $c = 0$. If $w/(1 - R(H - B)) \geq H - E[b|b \geq H - B]$, $F^* \leq 0$ and F^* decreases in w. If $w/(1 - R(H - B)) < H - E[b|b \geq H - B]$, $F^* > 0$.

Proof: See Appendix.

We, however, caution that this result does not rule out the possibility of corporate sanction. In particular, if the salary is sufficiently small, the firm is penalized when wrongdoing is detected.

6. Policy Implication

Our results imply a key deterrent to corporate misconduct is a worker's income stream, affected by his prospective compensation. Under the regulated economy, restricting compensation level weakens the deterrent effect of suspension on corporate misconduct. This result is contrastive to the current trend of executive compensation regulation such as DFA in the U.S. and CRD-IV in E.U. Under the decentralized economy, our results imply corporate sanction is essential for inducing a firm to internalize the social harm of business activity attributed to its worker's wrongdoing. In fact, it can be even negative if a worker's compensation is sufficiently large. Thus, if a regulator aims to achieve social optimum by raising a worker's compensation as much as possible, there is possibility that a regulator subsidizes a firm when a wrongdoing is detected, because a firm pays too much cost for its production without subsidy. This result is contrastive to the proposal of penalizing a firm upon the detection of wrongdoing by Arlen & Kraakman (1997).

Moreover, our analysis suggests the efficient level of monitoring, or probability of detecting a wrong, decreases in a worker's income stream and the intensity of suspension, at least when monitoring cost is relatively cheap. Under the modern system of internal governance and the incentive scheme that encourages self-reporting, monitoring cost is considered arguably cheap. Therefore, this result suggests excessive monitoring is unnecessary in the presence of high compensation. On the same ground, this result motivates the life-cycle theory of efficient monitoring: a younger firm should be monitored more frequently than an elder firm if a firm's compensation grows over time.

References

Albuquerque, R. & Cabral, L. & Guedes, J. (2019). Incentive Pay and Systemic Risk. *Review of Financial Studies, 32* (11), 4304-4342.

Arlen, J. (1994). Potentially Perverse Effect of Corporate Criminal Liability, *Journal of Legal Studies* 23 (2), 833-867.

Arlen, J. (2012). Corporate Criminal Liability: Theory and Evidence in Harel, A. and Hylton, N. K. (ed). *Research Handbook on the Economics of Criminal Law* 145-203, Edward Elgar, Northampton, MA, USA.

Arlen, J. The Potential Promise and Perils of Introducing Deferred Prosecution Agreements outside the U.S. (2020) in Søreide, T. and Makinwa, A. (ed). *Negotiated Settlements in Bribary Cases: A Principled Approach*, 156-199, Edward Elgar, Northampton, MA, USA.

Arlen, J. & Kraakman, R. (1997). Controlling Corporate Misconduct: An Analysis of Corporate Liability Regime. *New York University Law Review, 72* (4), 687-779.

Arlen, J. and Buell, S.W. (forthcoming) The Law of Corporate Investigations and Global Expansion of Corporate Criminal Enforcement, *Southern California Law Review, 93*.

Asai, K. (2016). Is Capping Executive Bonuses Useful? IMF Working Paper Series 13/4, International Monetary Fund, Washington.

Asai, K. (forthcoming). Limited Liability and Corporate Efficiency. *International Review of Law and Economics, 62*.

Bebchuk, L. A. & Kaplow, L. (1993). Optimal Sanctions and Differences in Individuals' Likelihood of Avoiding Detection. *International Review of Law and Economics, 13* (2), 217-224.

Beccaria, C. (1764). On Crime and Punishment in Bellamy, R (ed) (1995). *Beccaria On Crime and Punishment and Other Writings*, 1-113, Cambridge U.P., New York NY, USA.

Becker, G. (1965). A Theory of the Allocation of Time. *Economic Journal, 299*, 493-517.

Becker, G. (1968). Crime and Punishment: An Economic Approach. *Journal of Political Economy, 76* (2), 169-217.

Block, M. K. & Lind, R. C. (1975). An Economic Analysis of Crimes Punishable by Imprisonment. *Journal of Political Economy, 76* (2), 169-217.

Brandão Marques, L. & Oppers, S. E. & Ribeiro, I. A. & Asai, K. & Beauchamp, J. & Deb, P. & Duma, N. & Ehrentraud, J. & Khadarina, O. & Khan, A. & Pancorbo, A. & Pazarbaşioglu, C. & Singh, R. & Wuensch, O. (2014). Risk Taking By Banks: The Role of Governance and Executive Pay. In *Global Financial Stability Report 2014 October* (pp. 335-364). International Monetary Fund.

Bucy, P. (1991). Corporate Ethos: A Standard for Imposing Corporate Criminal Liability, *Minnesota Law Review, 75*, 1095-1184.

Coffee, J. C. (1981). “No Soul to Damn: No Body to Kick”: An Unscandalized Inquiry into the Problem of Corporate Punishment, *Michigan Law Rev*iew, *79*, 368-459.

DeYoung, R., Peng, E. Y., and Yan, M. (2013). Executive Compensation and Business Policy Choices at U.S. Commercial Banks. *Journal of Financial and Quantitative Analysis, 48* (1), 165–196.

Easterbrook, F. & Fischel, D. (1985). Limited Liability and the Corporation. *The University of Chicago Law Review, 52* (1), 89-117.

Edgerton, H. W. (1927). Corporate Criminal Liability, *Yale Law Journal*, *36*, 827-844.

Elkins. J. R. (1976). Corporations and the Criminal Law: An Uneasy Alliance. *Kentucky Law Journal, 65* (1), 73 -129.

Fahlenbrach, R. & Stulz, R. M. (2011). Bank CEO Incentives and the Credit Crisis. *Journal of Financial Economics, 99* (1), 11–26.

Fisse, B. (1983). Reconstructing Corporate Criminal Law: Deterrence, Retribution, Fault and Sanctions, *Southern California Law Review 56*, 1141-1246.

Fisse, B. and Braithwaite, J. (1993). *Corporations, Crime and Accountability*, Cambridge U.P., New York NY, USA.

Foucault, M. Sherdon, A. (trans). (1977). *Discipline & Punish: The Birth of the Prison*, Penguin books, London, UK.

Garland, D. (2001) *The Culture of Control: Crime and Social Order in Contemporary Society*, The University of Chicago Press, Chicago IL, USA.

Garoupa, F. (2002). The Theory of Optimal Law Enforcement. *Journal of Economic Surveys, 11* (3), 267-295.

Guala (2016), *Understanding Institutions: The Science and Philosophy of Living Together*, Princeton U.P., Princeton NJ, USA.

Hegel, G. W. F. (1820), Wood A. W. (ed) , Nisbet H. B. trans. (1995). *Elements of the Philosophy of Rights*, Cambridge U.P., Cambridge, UK.

Jakobs, G. (1997), *Norm, Preson, Geselschaft: Voruberlegungen zu einer Rechtsphilosophie*, Duncker & Humblot, Berlin, Germany.

Kaplow, L. & Shavell, S. (1994). Optimal Law Enforcement with Self-Reporting of Behavior. *Journal of Political Economy*, 102 (3), 583-606.

McAdams, R.H. (2015) *The Expressive Powers of Law: Theories and Limits,* Harvard U.P., Cambridge MA, USA.

Mueller, G. O. W. (1957), Mens Rea and the Corporation, *University of Pittsburgh Law Review*, *19*, 21-50.

Orland, L. (1980). Reflections on Corporate Crime: Law in Search of Theory and Scholarship, *American Criminal Law Review, 17* (501), 501 -520.

Polinsky, A. M. & Shavell, S. (1993). Should Employees Be Subject to Fines and Imprisonment Given the Existence of Corporate Liability? *International Review of Law and Economics*, *13* (3), 239-257.

Polinsky, A. M. & Shavell, S. (1999). On the Disutility and Discounting of Imprisonment and the Theory of Deterrence. *Journal of Legal Studies*, *28* (1), 1-16.

Rousseau, J.-J. (1755). Cress, D. A. (trans) (1910) Discourse on the Origin of Inequality, Dover Publications, Mineola NY, USA.

Shavell, S. (1987). The Optimal Use of Nonmonetary Sanctions as a Deterrent. *American Economic Review*, *77* (4), 584-592.

Winton, A. (1993). Limitation of Liability and the Ownership Structure of the Firm. *Journal of Finance*, *48* (2), 487-512.

Appendix

Preliminary analysis for stationary model: Because the value of an employee is time-invariant, equation (3) suggests

$$U = \lambda W + (1-\lambda)\beta U.$$

Hence,

$$W - U = \Gamma W,$$

where $\Gamma = (1-\lambda)(1-\beta)/(1-(1-\lambda)\beta)$. Then, $0 \leq \Gamma \leq 1$. Notice $\Gamma = 0$ when $\lambda = 1$, suggesting no imprisonment equates the value of being in and out of prison. Also, notice $\Gamma = 1$ when $\lambda = 0$, suggesting eternal imprisonment maximizes the relative value of being out of prison. Then, equation (2) suggests

$$(1-\beta)W = \int_{p(M)(\alpha w+\beta\Gamma W)}^{\infty} \big(b - p(M)(\alpha w + \beta\Gamma W\ )\big) r(b) db + w.$$

We start from analyzing how Γ is related to λ. We find:

$$d\Gamma/d\lambda < 0.$$

Next, we analyze how W is related to M and w, respectively. Recall:

$$(1-\beta)W = \int_{p(M)(\alpha w+\beta\Gamma W)}^{\infty} \big(b - p(M)(\alpha w + \beta\Gamma W\ )\big) r(b) db + w.$$

Then, we find from this equation:

$$(1-\beta)\partial W/\partial M = \int_{p(M)(\alpha w+\beta\Gamma W)}^{\infty} (-p'(M)(\alpha w + \beta\Gamma W\ ) - p(M)(\beta\Gamma\partial W/\partial M)) r(b) db,$$

$$(1-\beta)\partial W/\partial M + p(M)(\beta\Gamma\partial W/\partial M)\left(1-R(\tilde{b})\right) = -\left(1-R(\tilde{b})\right)p'(M)(\alpha w + \beta\Gamma W\ ),$$

$$\left(1-\beta+\beta\Gamma p(M)\left(1-R(\tilde{b})\right)\right)\partial W/\partial M = -\left(1-R(\tilde{b})\right)p'(M)(\alpha w + \beta\Gamma W\ ),$$

$$\partial W/\partial M = -\left(1-R(\tilde{b})\right)p'(M)(\alpha w + \beta\Gamma W\ )/\left(1-\beta+\beta\Gamma p(M)\left(1-R(\tilde{b})\right)\right) < 0.$$

Also, we find from this equation:

$$(1-\beta)dW/dw = \int_{\tilde{b}}^{\infty}(-p'(M)(\alpha w + \beta\Gamma W\ )dM/dw - p(M)(\alpha + \beta\Gamma dW/dw))r(b)db + 1,$$

$$\begin{aligned}&(1-\beta)dW/dw + p(M)(\beta\Gamma dW/dw)\left(1-R(\tilde{b})\right)\\ &\qquad = 1-\alpha p(M)\left(1-R(\tilde{b})\right) - p'(M)(\alpha w + \beta\Gamma W\ )\left(1-R(\tilde{b})\right)dM/dw,\end{aligned}$$

$$\begin{aligned}&\left(1-\beta+\beta\Gamma p(M)\left(1-R(\tilde{b})\right)\right)dW/dw\\ &\qquad = 1-\alpha p(M)\left(1-R(\tilde{b})\right) - p'(M)(\alpha w + \beta\Gamma W\ )\left(1-R(\tilde{b})\right)dM/dw,\end{aligned}$$

$$dW/dw = \left(1-\alpha p(M)\left(1-R(\tilde{b})\right)\right)/\left(1-\beta+\beta\Gamma p(M)\left(1-R(\tilde{b})\right)\right) + (\partial W/\partial M)(dM/dw).$$

Then,

$$dW/dw = \partial W/\partial w + (\partial W/\partial M)(dM/dw),$$

where

$$\partial W/\partial w = \left(1-\alpha p(M)\left(1-R(\tilde{b})\right)\right)/\left(1-\beta+\beta\Gamma p(M)\left(1-R(\tilde{b})\right)\right) \geq 0.$$

Given the above, the first order condition with respect to monitoring level is rewritten by

$$(p'(M)(\alpha w + \beta\Gamma W) + p(M)\beta\Gamma(\partial W/\partial M))r(\tilde{b})\,\tilde{\alpha} = c,$$

$$(1-\beta)p'(M)(\alpha w+\beta\Gamma W)r(\tilde{b})\,\tilde{a}/\left(1-\beta+\beta\Gamma p(M)\left(1-R(\tilde{b})\right)\right)=c.$$

From this analysis, we conclude $\tilde{a}^* \geq 0$.

Proof for Proposition 2: From an envelope theorem,

$$dS^*/dw=\sum_{t=0}^{\infty}\beta^t q^* p(M^*)(\alpha+\beta\Gamma\partial W/\partial w)\tilde{a}^* r(\tilde{b}^*)\geq 0.$$

Also,

$$dS^*/d\lambda=\sum_{t=0}^{\infty}\beta^t q^* p(M^*)(\beta W d\Gamma/d\lambda)\tilde{a}^* r(\tilde{b}^*)\leq 0.$$

Preliminary analysis for model with zero monitoring cost: Equation (2) together with equation (10) suggests

$$W_t=X+w_t+\beta W_{t+1},$$

where $X=\int_{H-B}^{\infty}(b-(H-B))r(b)db$. Using this expression, equation (3) suggests

$$\begin{aligned}(W_{t+1}-U_{t+1})-(W_t-U_t)\\ &=\Big((1-\lambda_{t+1})(X+w_{t+1})-(1-\lambda_t)(X+w_t)\\ &+\beta\big((1-\lambda_{t+1})(W_{t+2}-U_{t+2})-(1-\lambda_t)(W_{t+1}-U_{t+1})\big)\Big).\end{aligned}$$

From this relationship, we claim the following lemma.

Lemma 1: Suppose the social planner's problem under zero monitoring cost and $W_t-U_t\geq 0$ for any t. If $(W_{t+2}-U_{t+2})-(W_{t+1}-U_{t+1})\geq 0$, $w_{t+1}\geq w_t$, and $\lambda_{t+1}\leq\lambda_t$, then the following is true: $(W_{t+1} - U_{t+1}) - (W_t - U_t) \geq 0$. If $(W_{t+2} - U_{t+2}) - (W_{t+1} - U_{t+1}) \leq 0$, $w_{t+1} \leq w_t$, and $\lambda_{t+1} \geq \lambda_t$, then the following is true: $(W_{t+1} - U_{t+1}) - (W_t - U_t) \leq 0$.

Proof: Obvious from the above equation.

Also, equation (3) suggests

$$W_t - U_t = (1 - \lambda_t)(X + w_t) + \beta(1 - \lambda_t)(W_{t+1} - U_{t+1}).$$

From this relationship, we claim the following lemma.

Lemma 2: Suppose the social planner's problem under zero monitoring cost. If $W_{t+1} - U_{t+1} \geq 0$, $W_t - U_t \geq 0$.

Proof: Obvious from the above equation.

The value gap between being employed and unemployed is recursively determined, given the terminal condition:

$$W_T - U_T = (1 - \lambda_T)(X + w_T) + \beta(1 - \lambda_T)(W^* - U^*).$$

Proof for Proposition 3: Notice

$$W^* - U^* = \beta\Gamma W^* \geq 0.$$

Then, Lemma 2 suggests, for any t,

$$W_t - U_t \geq 0,$$

recursively.

Suppose $w_{t+1} \geq w_t$, and $\lambda_{t+1} \leq \lambda_t$ for any t. Then, because $W^* - U^* = \beta\Gamma W^* \geq 0$ and $X \geq 0$,

$$(W_{T+1} - U_{T+1}) - (W_T - U_T)$$
$$= (1 - \lambda_{T+1})(X + w_{T+1}) - (1 - \lambda_T)(X + w_T) + \beta(\lambda_T - \lambda_{T+1})(W^* - U^*) \geq 0.$$

Lemma 1 suggests, for any t,

$$\alpha w_t + \beta(W_{t+1} - U_{t+1}) \geq \alpha w_{t-1} + \beta(W_t - U_t),$$

recursively. Then, equation (10) suggests, for any t,

$$M_t^* \leq M_{t-1}^*.$$

The opposite is true if $w_{t+1} \leq w_t$, and $\lambda_{t+1} \geq \lambda_t$ for any t.

Under the stationary version, equation (10) is replaced by

$$\tilde{a} = H - p(M)(\alpha w + \beta\Gamma W) - B = 0.$$

Then the intersecting point of $\tilde{a}$ and the horizontal line at zero is M^*. Notice the second order condition requires

$$-p'(M)(\alpha w + \beta\Gamma W) - p(M)\beta\Gamma\partial W/\partial M \leq 0,$$

meaning $\tilde{a}$ is a decreasing function in M. Also,

$$-p(M)(\alpha + \beta\Gamma\,\partial W/\partial w) \leq 0,$$

meaning $\tilde{a}$ shifts down by an increase in w. The intersection hence decreases by an increase in w. From this observation, we conclude $dM^*/dw \leq 0$. On the other hand,

$$-p(M)\beta W d\Gamma/d\lambda \geq 0,$$

meaning $\tilde{a}$ shifts up by an increase in λ. The intersection hence increases by an increase in λ. From this observation, we conclude $dM^*/d\lambda \geq 0$.

Proof for Proposition 7: In equation (15), if $w/(1 - R(H - B)) \geq H - E[b|b \geq H - B]$, the right-hand-side is non-positive. As $p(M^*)$ decreases, the right-hand-side decreases. From Proposition 3, $dM^*/dw \leq 0$. Consequently, $dF^*/dw \leq 0$. On the other hand, if $w/(1 - R(H - B)) < H - E[b|b \geq H - B]$, the right-hand-side is positive. As $p(M^*)$ decreases, the right-hand-side increases. However, the right-hand-side also decreases, when w increases, for fixed $p(M^*)$. As these two effects oppose to each other, the effect of w is ambiguous.